

# The Canadian Decomposition Experience: From 10 to 54 Industries

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## ABSTRACT

Since the Canadian Government began reporting trends in energy intensity in industry using a decomposition model, data quality has increased significantly. This is due to the government's on-going commitment to better track and report energy consumption trends. As a result the analysis of industrial energy consumption has increased from 10 aggregate industrial sectors to 54 sub-industries.

General decomposition models decompose changes in energy consumption into changes in production level (the activity effect), changes in the relative share of production generated by sub-sectors (the structure effect) and changes in energy required per unit of production (the intensity effect). Combined the structure effect and intensity effect represent the change in the aggregate intensity for a sector. These models share a common problem i.e., the generation of residual terms. Often these terms are either ignored or regarded as interaction terms among the effects. In either case this leads to confusion for the reader or a potentially large estimation error. Recently a complete decomposition model was introduced which suggests equally distributing the residual terms among the effects associated with them. However this model assumes that the residual term generated by changes in both the activity effect and aggregate intensity effect should be equally divided between the intensity effect, structure effect and activity effect. As a result, this model will incorrectly estimate the impact of the three effects.

An update to this complete decomposition model is introduced here that solves this problem. In it, the residual terms are decomposed and distributed to the three effects based on the "jointly created and equally distributed" principle. Detailed Canadian industrial energy consumption data from 1990 to 1998 are used to compare the magnitude of the effects in a general decomposition model, the complete decomposition model and the updated complete decomposition model developed here.

## Introduction

The Office of Energy Efficiency (OEE) of Natural Resources Canada is committed to improving the tracking and reporting of trends in energy consumption in Canada. As part of this on-going commitment the OEE, working with other government agencies has increased the disaggregation of collected industrial energy data from 10 aggregate industrial sectors to 53 sub-industries. In an effort to better understand changes in energy consumption over time, the OEE has been reporting trends in energy efficiency for several years using a general decomposition model based on a Laspeyres index. Like other models of it's type the OEE decomposition model decomposes energy consumption into three effects, changes in the

level of economic activity (the activity effect), changes in the mix of industries (the structure effect) and changes in the technology level (the intensity effect<sup>1</sup>).

As part of its commitment to improved and develop better indicators of energy efficiency, the OEE searched for a decomposition model that would not be has confusing for the average reader and would more accurately track trends in energy efficiency. J.W. Sun presents a complete decomposition model in a recent article in the Journal of Energy Economics. The complete decomposition model attempts to split the interaction terms equally between those effects that generate them based on the principle “jointly created and equally distributed”.<sup>2</sup>

Though the methodology proposed by Sun does eliminate the interaction terms and the corresponding confusion, it violates the “jointly created and equally distributed” principle upon which it is based. The Sun model will tend to incorrectly estimate the total impact of changes in energy intensity, structure and activity. This paper will present an update to the Sun complete decomposition model that satisfies the need to eliminate the residual terms while adhering to the “jointly created equally distributed” principle. This will be followed by empirical results for Canadian industries using a general decomposition model, Sun’s complete decomposition model and the updated complete decomposition presented here.

## The Updated Complete Decomposition Model

The change in energy consumption is represented by the following equation:

$$\frac{E}{E_0} - 1 = \frac{A \sum_i \frac{A_i}{A} \frac{E_i}{A_i}}{A_0 \sum_i \frac{A_{i0}}{A_0} \frac{E_{i0}}{A_{i0}}} - 1$$

For simplicity:

$$\frac{E}{E_0} - 1 = \frac{A \sum_i a_i \Omega_i}{A_0 \sum_i a_{i0} \Omega_{i0}} - 1$$

Where  $A$  is activity,  $E$  is energy,  $a$  is industry  $i$ ’s share of total activity and  $\Omega$  is that industry’s energy intensity. The influence of each effect is determined by holding the other factors constant at base year levels.

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<sup>1</sup> In OEE publications this is often referred to as the “energy efficiency effect” to emphasize its difference from aggregate energy intensity (total energy to activity ratio). This variable is however influence by more than just changes in the technology level; it is also influenced by changes in the mix of production methods, the product mix and other factors such as weather.

<sup>2</sup> In their paper given at the APERC – Workshop on Energy Efficiency Indicators in Industry in September 1998 Eichhammer and Scholmann after comparing Sun’s complete decomposition model to other decomposition models, offered the following recommendation, “Taking into account all evaluation criteria the preference of the authors go to the recently proposed method by Sun (1998)...” (Eichhammer and Schlomann 1998)

$$\text{The activity effect} = \frac{A \sum_i a_{i0} \Omega_{i0}}{A_0 \sum_i a_{i0} \Omega_{i0}} - 1 \quad \text{or} \quad \left( \frac{A}{A_0} - 1 \right)$$

$$\text{The structure effect} = \frac{A_0 \sum_i a_i \Omega_{i0}}{A_0 \sum_i a_{i0} \Omega_{i0}} - 1 \quad \text{or} \quad \sum_i b_i \left( \frac{a_i}{a_{i0}} - 1 \right)$$

$$\text{The intensity effect} = \frac{A_0 \sum_i a_{i0} \Omega_i}{A_0 \sum_i a_{i0} \Omega_{i0}} - 1 \quad \text{or} \quad \sum_i b_i \left( \frac{\Omega_i}{\Omega_{i0}} - 1 \right)$$

Where  $b_i$  is industry  $i$ 's share of total energy consumption. The change in energy consumption can then be denoted by:

$$\frac{E}{E_0} - 1 = \left( \frac{A}{A_0} - 1 \right) + \sum_i b_i \left( \frac{a_i}{a_{i0}} - 1 \right) + \sum_i b_i \left( \frac{\Omega_i}{\Omega_{i0}} - 1 \right) + \delta + \varepsilon_1 + \varepsilon_2 + \varepsilon_3$$

This approach also generates residual terms often referred to as interaction terms as they are generated by the interaction between the three effects. For example,  $\varepsilon_2$  is the interaction term for the activity and intensity effects. If there were an increase in activity while energy intensity improved, the interaction effect would represent the change in energy consumption due to the improvement in energy intensity of the production of the new activity.

The interaction terms are expressed by the following equations.

$$\begin{aligned} \delta &= \sum_i b_i \left( \frac{a_i}{a_{i0}} - 1 \right) \left( \frac{\Omega_i}{\Omega_{i0}} - 1 \right) & \varepsilon_1 &= \left( \frac{A}{A_0} - 1 \right) \sum_i b_i \left( \frac{a_i}{a_{i0}} - 1 \right) \\ \varepsilon_2 &= \left( \frac{A}{A_0} - 1 \right) \sum_i b_i \left( \frac{\Omega_i}{\Omega_{i0}} - 1 \right) & \varepsilon_3 &= \left( \frac{A}{A_0} - 1 \right) \delta \end{aligned}$$

This can be confusing for the average reader as it does not provide a full allocation of the changes in energy consumption to the three factors (i.e. some of the impact of activity, structure and energy intensity are captured in the interaction terms). As in the example above, Canadian industries have seen improvements in energy intensity while activity levels have increased. When these effects are reported in the OEE's annual report, *Energy Efficiency Trends in Canada*, they are reported as though all other factors had remained constant. Thus the full impact of these effects is not captured; the total impact of increases in activity on energy consumption is actually less than what is reported because energy intensity has improved.

As Sun indicates in his paper, the purpose of proposing the complete decomposition model is to improve the reliability and accuracy of the decomposition model. The residual or

interaction terms are equally decomposed between the effects that generate them. A two-factor model is used here to express this principle.

Assume that  $V = xy$ , therefore the change in  $V$  can be expressed as:

$$\begin{aligned}\Delta V &= V - V_0 = xy - x_0y_0 \\ &= (x - x_0)y_0 + (y - y_0)x_0 + (x - x_0)(y - y_0) \\ &= y_0\Delta x + x_0\Delta y + \Delta x\Delta y\end{aligned}$$

In index form the base year values are equal to 1 so the previous equation may be written as:

$$\begin{aligned}\Delta V &= V - 1 \\ &= \Delta x + \Delta y + \Delta x\Delta y\end{aligned}$$

Sun proposes that the third term could be attributed to either  $x$  or  $y$  by equal right based on the principle "equally created equally distributed". The principle maintains that since the magnitude of the term is dependent on both the changes in  $x$  and  $y$  and should one of them go to zero the term disappears, each factor contributes equally to the residual term and as such it should be equally split among them. Therefore the contributions of these factors or their total effects on the change in  $V$  are expressed as:

$$\text{The total } x \text{ effect} = \Delta x + \frac{1}{2}\Delta x\Delta y$$

$$\text{The total } y \text{ effect} = \Delta y + \frac{1}{2}\Delta x\Delta y$$

Graphically this is represented by Figure 1.

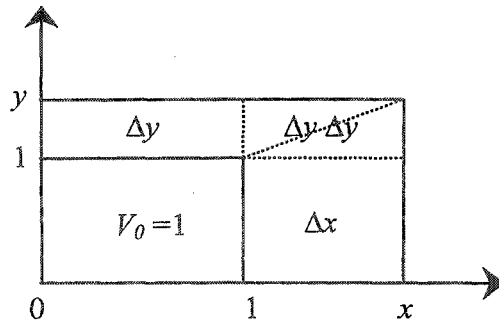


Figure 1. Total  $x$  and  $y$  Effects on  $V$

Sun then describes a three factor system, wherein  $V = xyz$ . The contributions in index form of each factor to the total change in  $V$  are represented by the following formulas.

$$\text{The total } x \text{ effect} = \Delta x + \frac{1}{2}\Delta x\Delta y + \frac{1}{2}\Delta x\Delta z + \frac{1}{3}\Delta x\Delta y\Delta z$$

$$\text{The total } y \text{ effect} = \Delta y + \frac{1}{2}\Delta x\Delta y + \frac{1}{2}\Delta y\Delta z + \frac{1}{3}\Delta x\Delta y\Delta z$$

$$\text{The total } z \text{ effect} = \Delta z + \frac{1}{2} \Delta y \Delta z + \frac{1}{2} \Delta x \Delta z + \frac{1}{3} \Delta x \Delta y \Delta z$$

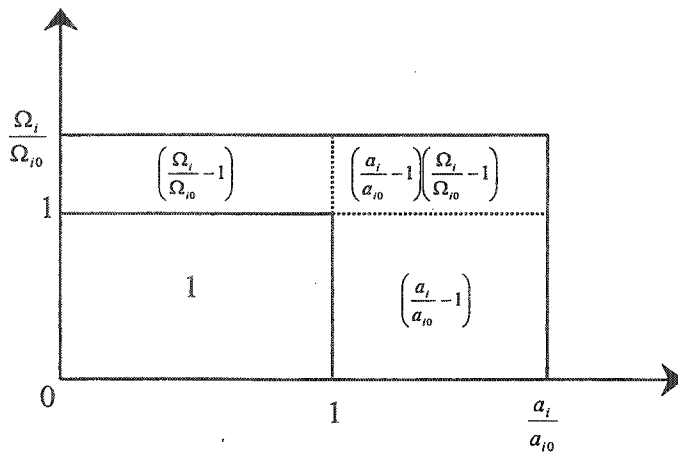
This portion of the model holds true, when all three variables interact at the same level of disaggregation. This is not the case in the energy model used to decompose changes in energy consumption into changes in activity, structure and energy intensity. In the energy model the interaction between the structure effect and the intensity effect occur at the sub-industry level and not the total industry level as Sun has assumed. Here are the three total effects using Sun's methodology.

$$\text{Total Activity effect} = \left( \frac{A}{A_0} - 1 \right) + \frac{1}{2} \varepsilon_1 + \frac{1}{2} \varepsilon_2 + \frac{1}{3} \varepsilon_3$$

$$\text{Total Structure effect} = \sum_i b_i \left( \frac{a_i}{a_{i0}} - 1 \right) + \frac{1}{2} \varepsilon_1 + \frac{1}{2} \delta + \frac{1}{3} \varepsilon_3$$

$$\text{Total Intensity effect} = \sum_i b_i \left( \frac{\Omega_i}{\Omega_{i0}} - 1 \right) + \frac{1}{2} \varepsilon_2 + \frac{1}{2} \delta + \frac{1}{3} \varepsilon_3$$

As mentioned above this assumes that  $\delta$  is equal to the product of the structure effect and the intensity effect. In fact  $\delta$  is the sum, over all industries, of the product of the change in intensity and the change in the share of total activity for each industry weighted by that industry's share of total energy consumption. Figure 2 depicts the "precursor" to  $\delta$ , before it is weighted by that industry's share of total energy consumption.



**Figure 2. Index of Energy Intensity vs. Index Share of Total Activity of Industry  $i$**

When this is weighted by each industry's share of total energy consumption and summed across industries, it is equal to the change in aggregate energy intensity for the entire industrial sector (i.e., the change in total industrial energy consumption over total industrial activity).

$$\frac{\frac{E}{A}}{\frac{E_0}{A_0}} - 1 = \sum_i b_i \left( \frac{a_i}{a_{i0}} - 1 \right) + \sum_i b_i \left( \frac{\Omega_i}{\Omega_{i0}} - 1 \right) + \sum_i b_i \left( \frac{a_i}{a_{i0}} - 1 \right) \left( \frac{\Omega_i}{\Omega_{i0}} - 1 \right)$$

Sun's methodology results in a miscalculation of  $\delta$ , it assumes that the interaction of the structure effect and intensity effect occurs at the sector level thus assuming:

$$\delta = \sum_i b_i \left( \frac{a_i}{a_{i0}} - 1 \right) \sum_i b_i \left( \frac{\Omega_i}{\Omega_{i0}} - 1 \right)$$

while it is correctly expressed as:

$$\delta = \sum_i b_i \left( \frac{a_i}{a_{i0}} - 1 \right) \left( \frac{\Omega_i}{\Omega_{i0}} - 1 \right)$$

Graphically total energy is decomposed into the three effects and the interaction terms in Figure 3.

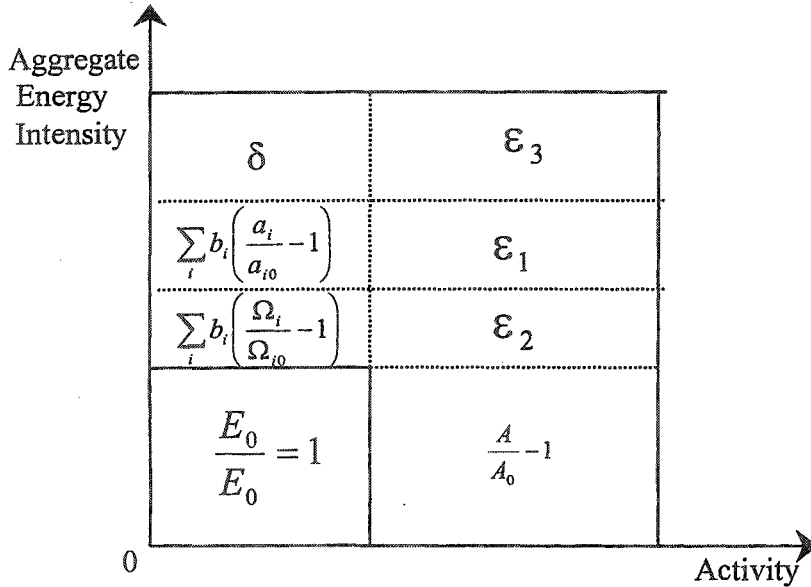


Figure 3. Total Energy Consumption Decomposed into its Components

In the updated complete decomposition model as in Sun's complete decomposition model  $\delta$ , though calculated differently, is equally distributed between the structure and intensity effects. While  $\epsilon_1$  and  $\epsilon_2$  are equally distributed between the structure and activity effects, and intensity and activity effects respectively. A further update to Sun's methodology occurs in the distribution of  $\epsilon_3$  the interaction between  $\delta$  and the activity effect.

As  $\delta$  is equally created by both structure and intensity effects and it jointly creates  $\varepsilon_3$  with the activity effect,  $\varepsilon_3$  should be distributed such that one half of it is distributed to the activity effect while one quarter is distributed to both the structure and activity effects. Thus the “total” effects on energy consumption are expressed as:

$$\text{Total Activity effect} = \left( \frac{A}{A_0} - 1 \right) + \frac{1}{2} \varepsilon_1 + \frac{1}{2} \varepsilon_2 + \frac{1}{2} \varepsilon_3$$

$$\text{Total Structure effect} = \sum_i b_i \left( \frac{a_i}{a_{i0}} - 1 \right) + \frac{1}{2} \varepsilon_1 + \frac{1}{2} \delta + \frac{1}{4} \varepsilon_3$$

$$\text{Total Intensity effect} = \sum_i b_i \left( \frac{\Omega_i}{\Omega_{i0}} - 1 \right) + \frac{1}{2} \varepsilon_2 + \frac{1}{2} \delta + \frac{1}{4} \varepsilon_3$$

Graphically the total effects are represented in Figure 4.

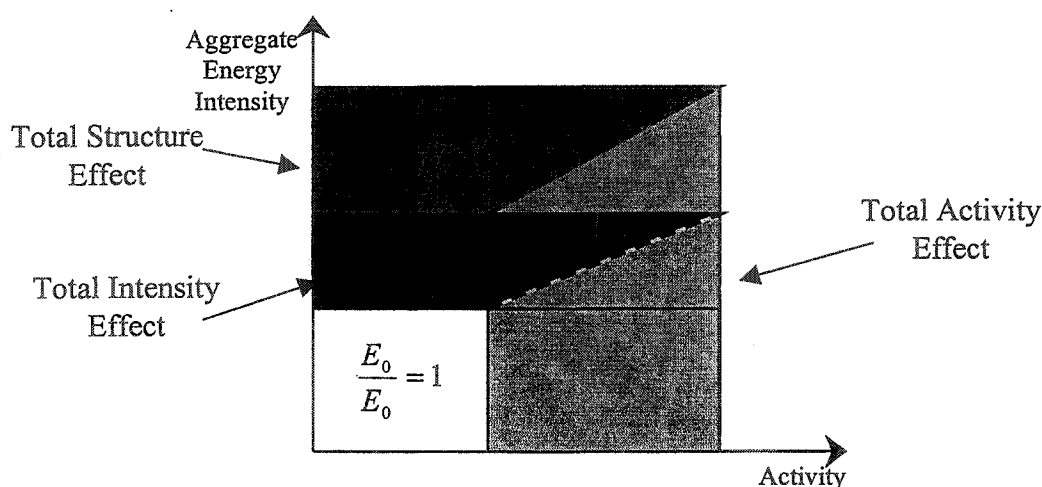


Figure 4. Total Energy Consumption Decomposed into the Three Total Effects

## Decomposition of Industrial Energy Use for Canada 1990 to 1998

Tables 1 through 4 provide an empirical comparison of the results of a general decomposition model, Sun's complete decomposition model and the updated complete decomposition model as changes in petajoules of energy consumption. These results were produced using energy and activity data for 53 industries. The activity variable used, is a composite of real gross domestic product, gross output and physical units of production (see *Energy Efficiency Trends in Canada 1990 to 1998* for a more detailed explanation of this variable).

**Table 1. General Decomposition Model Results (petajoules)**

	1990	1991	1992	1993	1994	1995	1996	1997	1998
<b>Total Energy Demand</b>	2754.66	2701.04	2723.04	2748.03	2911.50	2973.55	3057.53	3057.22	3003.99
<b>Activity Effect</b>	0.00	-139.21	-130.61	-42.03	142.06	225.84	265.17	485.83	602.91
<b>Structure Effect</b>	0.00	106.30	153.86	158.29	151.54	164.87	150.98	63.46	0.62
<b>Intensity Effect</b>	0.00	-5.05	-35.06	-119.41	-117.78	-139.41	-91.23	-192.73	-261.24
$\delta$	0.00	-11.10	-14.89	-2.93	-19.70	-31.87	-25.36	-26.52	-29.46
$\epsilon_1$	0.00	-5.37	-7.29	-2.42	7.81	13.52	14.53	11.19	0.13
$\epsilon_2$	0.00	0.26	1.66	1.82	-6.07	-11.43	-8.78	-33.99	-57.18
$\epsilon_3$	0.00	0.56	0.71	0.04	-1.02	-2.61	-2.44	-4.68	-6.45

The earlier 1990s were marked by a recession in Canada. The effect of which is noted in the negative impact of activity on energy consumption in this period in Table 1. Since the earlier 1990s activity has been increasing thus increasing the demand for energy. In 1998 activity continued to put upward pressure on energy consumption while the structure effect and the intensity effect partially offset this pressure, indicating an increase in the share of less energy intensive industries and a general improvement in energy intensity for the industrial sector.

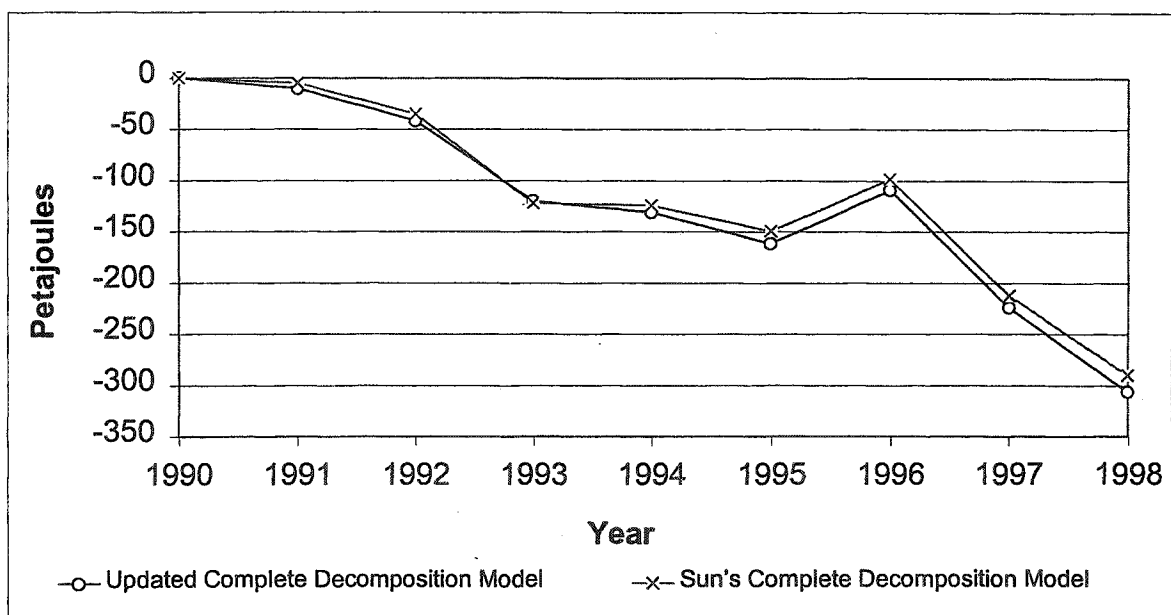
**Table 2. Sun's Complete Decomposition Model Results (petajoules)**

	1990	1991	1992	1993	1994	1995	1996	1997	1998
<b>Total Activity Effect</b>	0.00	-141.77	-133.39	-42.29	142.82	226.65	267.88	474.17	574.39
<b>Total Structure Effect</b>	0.00	103.52	149.27	153.68	152.10	167.22	155.59	66.57	0.65
<b>Total Intensity Effect</b>	0.00	-5.02	-35.18	-121.89	-124.17	-149.52	-98.28	-212.20	-289.87

**Table 3. Updated Complete Decomposition Model Results (petajoules)**

	1990	1991	1992	1993	1994	1995	1996	1997	1998
<b>Total Activity Effect</b>	0.00	-141.49	-133.07	-42.31	142.42	225.57	266.82	472.09	571.17
<b>Total Structure Effect</b>	0.00	98.21	142.95	155.63	145.35	155.03	144.96	54.62	-15.66
<b>Total Intensity Effect</b>	0.00	-10.33	-41.50	-119.95	-130.92	-161.71	-108.91	-224.15	-306.17





**Figure 5. Comparing Total Intensity Effect from Sun's and the Update Complete Decomposition Models**

The increase in energy intensity in 1996 is thought to be the result of that year's weather. In Canada, 1996 was significantly colder than the rest of the 1990's, which would result in an increased demand for heating but would not have an impact on production levels thus increasing energy intensity.

**Table 4. Difference Between Sun's and the Updated Complete Decomposition Models (petajoules)**

	1990	1991	1992	1993	1994	1995	1996	1997	1998
<b>Total Activity Effect</b>	0.00	0.28	0.32	-0.01	-0.40	-1.08	-1.06	-2.08	-3.22
<b>Total Structure Effect</b>	0.00	-5.31	-6.32	1.94	-6.75	-12.19	-10.63	-11.95	-16.31
<b>Total Intensity Effect</b>	0.00	-5.32	-6.32	1.94	-6.75	-12.19	-10.63	-11.95	-16.31

The differences between Sun's model and the updated model can be significant. In 1998, for example, there is significant difference between the total structure effect and intensity effect in the models. In fact the total structure effect is negative in the updated model and positive in the result from Sun's model, as is the structure effect in the general model. The updated model is more accurate than Sun's model as it reflects the fact that though there has been a shift towards what were more energy intensive industries these industries have, since 1990, improved their energy intensity significantly enough to decrease overall energy consumption.

Note that because of the miscalculation of  $\delta$  in Sun's methodology, the base year energy consumption plus the three total effects in any given year do not sum to the total energy consumption for that year. Table 5 presents  $\delta$  from both models.

**Table 5.  $\delta$  In Both The Updated and Sun's Complete Decomposition Models**

	1990	1991	1992	1993	1994	1995	1996	1997	1998
$\delta$ Updated Model	0.00	11.10	-14.89	-2.93	-19.70	-31.87	-25.36	-26.52	-29.46
$\delta$ Sun's Model	0.00	-0.19	-1.94	-6.85	-6.85	-9.01	-5.55	-4.93	-0.06

## Conclusions

The OEE in its on-going commitment to improve tracking and reporting of energy consumption and efficiency trends in Canada has improved the disaggregation of industrial energy data. Further to this commitment the OEE has developed and adopted a new model to track trends in energy efficiency based on Sun's complete decomposition model. This updated model more accurately tracks change in energy intensity and should alleviate confusion for the average reader.

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