Decomposing Daily Energy Load Series into Weather-Sensitive and Non-Weather-Sensitive Components Using Multivariate State Space Time Series Analysis

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A perennial concern for electricity demand forecasting is how best to allocate a forecast of total annual demand to a pattern of daily loads and to 24-hour load profiles for typical and peak-demand days. Patterns of electricity demand, both long-run seasonal ones and short-run diurnal ones, depend in complex ways on behavioral and economic variables as well as on regional climatic features and weather conditions. This paper develops a new way to allocate an annual demand forecast to daily consumption, and suggests a natural way to extend the method for generating daily load profiles. The idea is first to estimate a weather-based regression model, called the weather-sensitive (WS) component, that captures predictable features of energy demand such as the need for lighting, heating and cooling. Such a model leaves unexplained a serially-correlated residual or non-weather-sensitive (NWS) component which is modeled using state space time series analysis. In an application using SMUD system load data, the WS regression models accounted for 71 to 91 percent of the total daily variation, and accurately identified peak days in out-of-sample prediction. The models generally fell short of extreme values, however, systematically underpredicting high loads and overpredicting low loads. Addition of the estimated NWS component models to the WS models reduced the size of but did not eliminate the systematic errors of the WS regression models.

Introduction

Energy planning relies on a variety of forecasting methods applied at different levels of aggregation over end use, rate class and time. One perennial concern for electricity demand forecasting is how best to allocate a total annual load forecast to a pattern of daily loads and to 24-hour load profiles for typical and peak-demand days. These patterns of electricity demand, over the course of the year and over the course of the day, depend in complex ways on behavioral variables as well as on regional climatic features and weather conditions. This paper proposes a new procedure for allocating an annual demand forecast to a daily load pattern. The idea is to decompose energy load into weather-sensitive (WS) and non-weather-sensitive (NWS) components using multiple linear regression and multivariate state space time series analysis. Then, given a prediction E_i of total energy consumption for year i, plus a model of typical or expected weather variation based on historical data, the WS and NWS component models can be used to generate a pattern of daily deviations from the annual mean daily load. Thus, for day t of year i the estimate E_{it} of daily load would be

$$E_{it} = (E_i/365) + WS_{it} + NWS_{it} \qquad (1)$$

where WS_{it} is the deviation from annual mean load due to weather conditions and day length and is based on an estimated regression model, and NWS_{it} is the unexplained pattern of residual variation captured by a state space time series model.

For any given load series, weather dependence is modeled first to capture the structure of the WS component. The modeling technique is multiple linear regression, possibly including lagged weather variables. Next, the highly auto-correlated regression residuals for several related load series (total daily load, and loads at maximum and minimum hours) are pooled to form a multivariate time series, which is modeled to capture the structure of the NWS component. The modeling technique here is the "state space time series" approach developed by Aoki, henceforth referred to as Aoki-SSTS (see Aoki [1987] and Aoki and Havenner [1991]). The Aoki-SSTS approach is useful in the present context because it exploits the correlations among related individual series to generate improved forecasts. In this paper the WS and NWS models are estimated using daily SMUD system load data for 1983 to 1988, then tested out-of-sample on forecasts of 1989 energy loads.

Analytical Approach

The hourly energy load series for a full year contains at least two levels of dynamics: long-run dynamics due to seasonal variation in energy demand, and short-run dynamics due to diurnal variation in demand. Our approach is to model the long-run dynamics directly, and then to extract the short-run dynamics only indirectly via daily models for each hour of the day. In other words, we estimate models for 27 series which together describe a full year's energy demand: three daily summary models (total daily load, and load at maximum and minimum hours) and 24 hourly models for all the hours of the day. Yearly-to-daily allocation is then accomplished using the three daily summary models, while 24-hour load profiles are constructed using the 24 hourly models.[1] This paper discusses only the yearly-to-daily allocation procedure.

The Structure of the Model

Let Y_{it} represent the value of an energy load variable on day t of year i. For example, Y_{it} may be the total electricity consumed in each 24-hour period. Following the forecasting strategy embodied in equation (1) above, the model to be estimated features energy load expressed as a deviation from its annual average:

$$Y_{it}-\overline{Y}_i=\beta X_{it}+\nu_{it} \qquad (2)$$

where βX_{it} represents the WS component, X_{it} contains the values of the weather variables, and ν_{it} is the NWS component whose structure is characterized in state space form by the relations

$$\nu_{it}=CZ_{it}+e_{it} \qquad (3)$$

$$Z_{i,t+1}=AZ_{it}+Be_{it} \qquad (4)$$

where e_{it} is random white noise, A, B and C are coefficient matrices to be estimated, and the Z_{it} are the unobservable states. Equation (3) is called the observation equation; equation (4) is called the state equation.

For the daily analysis three Y_{it} series are modeled: total daily load, and load at the maximum and minimum hours. The regression model of equation (2) is estimated independently for each of the load variables using ordinary least squares.[2] As one would expect, each residual series exhibits a high degree of autocorrelation, as can be seen in the pattern of residuals shown in Figure 1 below.

The three residual series are pooled to form the three-dimensional vector observation ν_{it} specified in equation (3). The function of the states Z_{it} is to carry the lagged information on ν_{it} needed for forecasting the NWS component.[3] Given estimates of the coefficient matrices A, B and C we compute the values of Z_{it} recursively, using equation (4), in order to generate forecasts of ν_{it}.

The Weather-Sensitive (WS) Component Model: Specification, Interpretation and Evaluation

Currently accepted practice in energy-load modeling, augmented by insights gained from a recent study of weather-energy relationships by one of the present authors, provided the basis for specifying the WS component model adopted here.[4] Since each load variable Y_{it} was modeled for the full year, we included weather variables that capture both the effects of summer weather on air conditioner use, and the effects of winter weather on electric heating appliances. In estimating the regression models the dependent load variable was expressed as a deviation from annual mean. This specification was used to avoid having to accommodate long-run demographic factors in the models.

Summer weather is represented by a variable called THIDAY, defined by the formula

$$THIDAY_{it}=\sum_{j=1}^{24}\max\,(THI_{itj}-68,0) \qquad (5)$$

where THI_{itj} is the hourly temperature-humidity index, a weighted sum of drybulb and dewpoint temperature. Two models are estimated in this study, one including THIDAY values for the current day and two previous days, the other including only the current day value. The lag structure of the first model follows the example of peak models currently used by the California Energy Commission; see CEC [1991]. The same report also discusses the use of the 68-degree base. The second model, with no lagged weather variables, is preferable for generating residuals to estimate the NWS component, since the SSTS approach produces superior forecasts when all the dynamic or lagged information is contained in the residual term. As we will see, the two models give quite similar regression results.

Winter weather is represented by a variable called HDDAY, based on the concept of heating degree days and defined by the formula

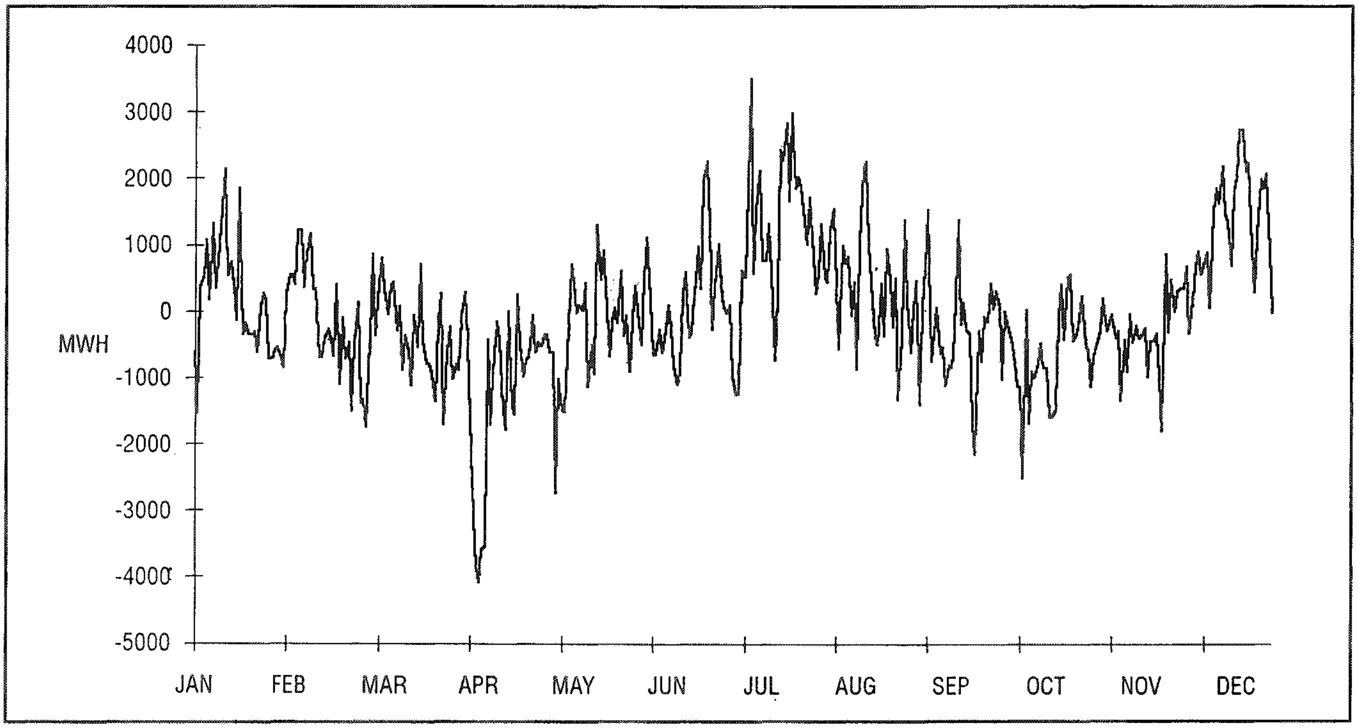

Figure 1. Residuals from 1989 Out-of-Sample Forecast of WS Component Model - Total Daily Load

$$HDDAY_{it}=\sum_{j=1}^{24} \max\ (55-TEMP_{itj},0) \qquad (6)$$

where $TEMP_{itj}$ is the hourly temperature. Again, one model includes HDDAY values for the current day and two previous days, while the other includes only the current day. The 55-degree base in (6) is a downward adjustment from the standard 65-degree base, to account for the increased energy efficiency of buildings constructed since the standard was first established. The lower base also reflects the fact that in most commercial and industrial buildings internal heat is being generated by normal activity, thus reducing the need for additional heating at any given outdoor temperature.

The WS component models contain two other variables that are not truly weather variables but which capture predictable aspects of energy demand. One is photoperiod or day length. This is simply the length of time in hours from sunrise to sunset, and is obtained by a simple calculation based on julian date and latitude.[5] It is included to capture the seasonal affect of ambient light. The other non-weather variable is a dummy which equals one for weekends and holidays.

In constructing the WS component models, we anticipated that all the weather variables would have positive coefficients. In the summer months the HDDAY values will be zero or close to zero, and the THIDAY variables will capture the effects of high temperature and humidity on air conditioner use. In the winter the roles of the variables will be reversed: THIDAY values will be zero, and the HDDAY variables will capture the effects of low temperatures on demand for heating energy. The only coefficients expected to be negative are those on the weekend/holiday dummy and on day length: energy use will generally decline on weekends and holidays in an urban/office environment like Sacramento, and demand for lighting will rise as the daylight hours grow shorter. Assuming the models contain adequate weather variables, the day length variable should represent a pure daylight effect uncontaminated by the effect of high summer temperatures on air-conditioning demand.

Once the WS component models are estimated, they can be subjected to some critical evaluation. In particular, given the motivating problem of how to allocate an annual load forecast to peak load days, it is important to see how the models perform in an out-of-sample prediction test. Once the coefficients are estimated using 1983-88 data, the models are run on 1989 weather data and used to predict daily sums, maxima and minima. If the modeling strategy is working as intended, the models should be able to identify the peak load days of 1989.

The Non-Weather Sensitive (NWS) Component Model

The motivation for state space modeling of a non-weather-sensitive component arises from three observations. First, regression models of energy load typically exhibit highly autocorrelated residuals. Although this fact does not bias the coefficient estimates, it does mean that forecasts will systematically be too high at some times of the year and too low at others. Second, regression models for different dependent variables yield residuals that are correlated across series. Since a major problem of autocorrelation is a loss of efficiency in estimation, even for large samples, it is clearly desirable to increase efficiency by exploiting relationships among the different dependent variables being modeled. Finally, it is not at all clear how best to model non-weather-sensitive aspects of energy demand. Theoretical modeling of the interaction of seasonal cycles with short-run demographic and economic factors is highly susceptible to misspecification errors. Thus it becomes practical to take a "data-based" approach to model specification, that is, to let the data itself reveal the implicit structure by which current observations depend on previous history.

Results

In what follows we report first on the performance of the WS models alone, prior to any attempt to model the time series properties of the residuals. The WS component models perform well in identifying peak load days, particularly summer peaks. A problem that arises in identifying winter peaks is that ornamental Christmas lighting causes a rise in electricity use that is not related to weather conditions. A general property that becomes obvious in examining the results of the WS models is their tendency to underpredict the values of peak loads. The systematic pattern to these errors is what the NWS state space model is intended to capture, and these results are reported in the second subsection.

The WS Component Models

Estimated regression equations for daily sum and daily maximum and minimum hourly load are presented in Table 1. Model A, in the top half of the table, contains lagged weather variables, while Model B in the bottom half contains only current weather variables.

The results may be summarized as follows:

(1) Models A and B perform quite similarly.

(2) The weekend/holiday dummy has a significant negative effect.

(3) Day length has a significant negative coefficient, supporting the intuition that this variable should capture a pure daylight effect once the seasonal weather effects are adequately represented in the model.

(4) All weather variables have significant positive effects. (The one exception is the coefficient of $HDDAY_{t-2}$ in the MIN model, which is not significant.)

(5) Adjusted R^2 values are uniformly quite high.

(6) The Durbin-Watson statistics are small in magnitude and highly significant, indicating strong positive autocorrelation in the residual series.

These results are all in accord with the expectations outlined in the previous section.

Next, the models are evaluated for their ability to predict peak days. At this point we abandon Model A and use only Model B for the rest of this paper. As mentioned above, the dynamics of the process are best left in the residuals that are input into the state space time series procedure. Weather data for 1989 are used to generate forecasts of the three daily load variables, and these forecasts are compared with actual 1989 load values. Days are ranked from highest to lowest, separately for summer (May through September) and winter (November through March), according to the values of four variables, namely, actual and forecast daily sum and daily maximum. Table 2 lists the top-ranked days according to actual total and actual maximum load, by season.

The results may be summarized as follows:

(1) For the summer season, the WS models identify five of the six top-ranked days, although there are some disagreements about the order among the top days.

(2) The summer forecast values are consistently below the actual values, indicating a tendency for the WS models to underpredict high load values.

(3) The four variables perform less consistently in winter than in summer. Note, however, that the data itself is less consistent, as reflected in the different ranks assigned by the two actual load variables. Still, the models successfully detect the five top days as ranked by either of the actual variables.

Table 1. Estimates of Regression Models for the Weather-Sensitive (WS) Component of Energy Load. Estimated coefficients (t-values).

	Daily Sum	Daily Max	Daily Min	Mean of Variable
Model A				
Intercept	2025 (9.7)	82.7 (5.4)	61:4 (9.9)	
WEEKEND	-2334 (-53)	-127 (-39)	-16.1 (-12)	.312
DAY LENGTH	-350 (-21)	-21.7 (-18)	-9.50 (-19)	12.1
$THIDAY_t$	49.9 (46)	4.54 (57)	.217 (6.8)	27.4
$THIDAY_{t-1}$	16.6 (11)	1.02 (9.3)	.509 (11)	27.4
$THIDAY_{t-2}$	5.53 (5.2)	.578 (7.3)	.165 (5.2)	27.4
$HDDAY_t$	8.72 (16)	.472 (11)	.358 (22)	71.1
$HDDAY_{t-1}$	3.23 (4.4)	.131 (2.4)	.107 (4.9)	71.1
$HDDAY_{t-2}$	1.57 (2.8)	.118 (2.9)	.013 (.78)	70.9
No. Obs.	2157	2157	2157	
Adj. R^2	0.889	0.908	0.772	
D-W	0.641	0.710	0.588	
Model B				
Intercept	1931 (8.5)	66.6 (4.0)	56.1 (8.3)	
WEEKEND	-2381 (-48)	-131 (-36)	-17.6 (-12)	.312
DAY LENGTH	-324 (-18)	-19.0 (-14)	-8.52 (-16)	12.1
$THIDAY_t$	67.0 (101)	5.74 (118)	.738 (37)	27.4
$HDDAY_t$	12.5 (42)	.656 (30)	.450 (51)	71.1
No. Obs.	2159	2159	2159	
Adj. R2	0.862	0.886	0.715	
D-W	0.786	0.799	0.722	

(4) The two models tend to underpredict load on peak days, as was observed for the summer season.

Overall, the WS component models perform well in detecting peak days in summer and in winter. One improvement in winter may be to add a Christmas dummy to capture the effects of ornamental lighting on electricity consumption. The most obvious shortcoming of the WS models is the clear tendency to overpredict at certain times and to underpredict at others. We argue, however, that these tendencies are not so much shortcomings of the WS models as evidence of a dynamic process that can be captured by the SSTS model. In other words, the residuals from the WS regression models follow a pattern which adversely affects the unaided WS models' forecasting ability, but which contains the dynamic structure that the NWS model can capture. A typical example of the pattern of residuals is shown in Figure 1, the 1989 residuals from the out-of-sample forecast of the total daily load WS component model. This pattern suggests a six-month cycle, with peaks coming at the beginning and the middle of the year.

The NWS Component Model

A broader view of the cyclical pattern in the WS model residuals is provided by Figure 2, which covers all the years 1983 through 1989. Daily residuals from the total daily load model are averaged by month for two reasons. First, averaging allows a more global view of the

Table 2. Performance of WS Regression Models in Forecasting Peak Load Days. Models estimated using 1983-88 data, applied and tested out-of-sample on 1989 forecast year. Entries indicate the ranks assigned to dates by each variable, and (values of the variables).

	Actual Daily Sum	Forecast Daily Sum	Actual Max Load	Forecast Max Load
Summer Peaks				
18 July	2 (32790)	2 (30366)	1 (1998)	2 (1889)
7 July	1 (32845)	1 (30962)	2 (1969)	1 (1942)
19 July	3 (32004)	3 (29724)	3 (1950)	3 (1834)
20 July	4 (30878)	7 (28044)	4 (1918)	7 (1689)
6 July	5 (30081)	5 (29571)	5 (1861)	5 (1823)
17 July	6 (29959)	4 (29710)	6 (1846)	4 (1833)
Winter Peaks				
6 February	3 (29158)	2 (27939)	1 (1523)	2 (1427)
7 February	7 (28302)	1 (27952)	2 (1487)	1 (1428)
21 December	1 (29177)	10 (26447)	3 (1485)	10 (1350)
20 December	2 (29164)	11 (26421)	4 (1481)	11 (1349)
8 February	12 (27856)	3 (26981)	5 (1468)	3 (1376)
19 December	4 (28899)	4 (26869)	7.5 (1458)	4 (1373)
18 December	5 (28641)	5 (26805)	9 (1453)	5 (1369)

seven-year period being modeled, without the distraction of day-to-day noise. Second, these monthly means are the actual data to which the Aoki-SSTS procedure is applied in this phase of the development of our procedure. The last twelve points in Figure 2 are the monthly residuals from the 1989 out-of-sample forecast of the WS model. The NWS models are estimated on the 1983-88 data, and then forecast for 1989 as we did with the WS models.

Note how the data of Figure 2 repeat the two-peaks-per-year pattern observed in Figure 1. The presence of a cyclical pattern in the WS residuals suggests that there are as yet unexplained dynamics that can be captured by a well-specified time series model. The details of the Aoki state space approach will not be discussed here; the reader is referred to Aoki [1987] and Aoki and Havenner [1991] for a full treatment that covers both the rationale behind the state space specification and the mechanics of estimating the coefficient matrices A, B and C of equations (3) and (4).

The first way to evaluate the success of the NWS models is through the reduction in variance of the residuals. As Table 3 indicates, the Aoki-SSTS estimates of the NWS component models accounted for 42 to 46 percent of the variance of the residuals generated by the WS regression models.

A second evaluation of the NWS state space model is illustrated in Figure 3, which shows the out-of-sample forecast for 1989. Actual WS residuals, averaged by month, are shown beside the corresponding monthly forecasts of the NWS model. The following observations should be noted:

(1) In all but one month (February) the NWS estimates have the correct sign. The state space model is clearly capturing the six-month cycle that appears in Figures 1 and 2.

(2) The magnitudes of the NWS model estimates need improvement. The final residuals, i.e., the differences between the black and white bars in Figure 3, appear to be serially correlated but do not seem to systematically overpredict or underpredict. The questions raised by these observations will be the focus of further research.

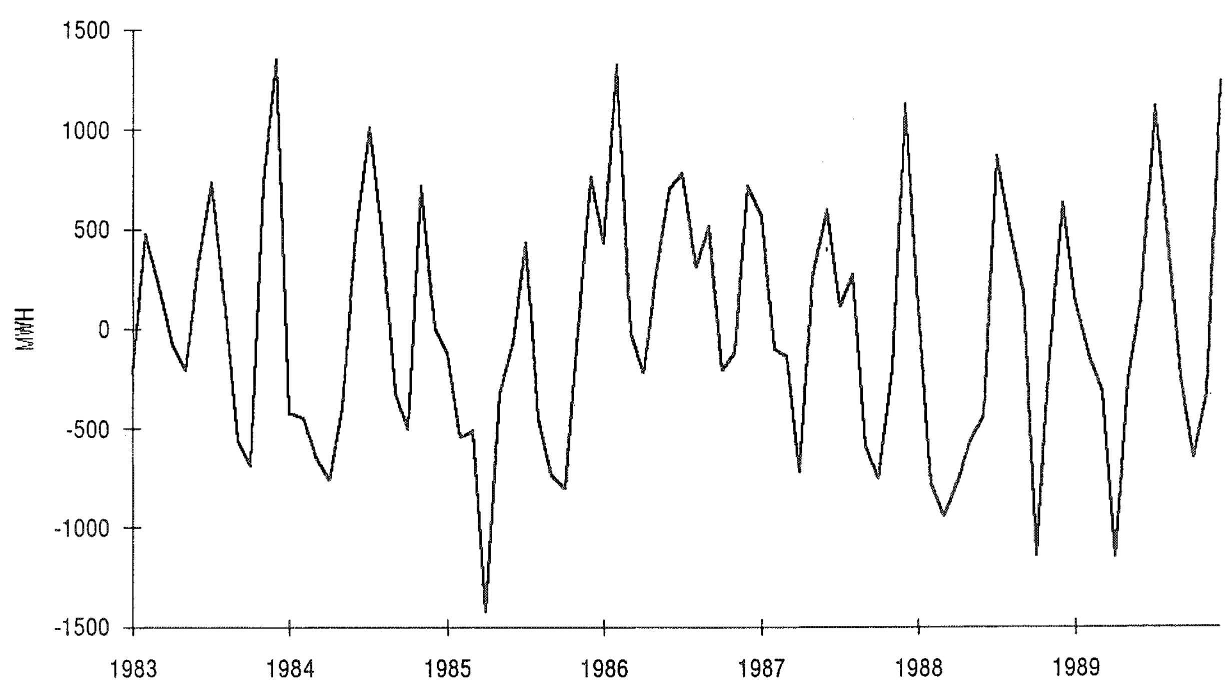

Figure 2. 1983-1989 Residuals from WS Component Model of Total Daily Load - Monthly Averages

A final evaluation of the full procedure, an out-of-sample test of the WS and NWS components combined, is shown in Figures 4 and 5. Figure 4 shows total daily load estimates for July 1989, the month of the summer peak, and Figure 5 shows total daily load estimates for February 1989, the month of the winter peak. In both cases the WS component is generated by Model B of Table 1. This is a quite severe test of the procedure, for we are trying to predict actual daily loads for a full year beyond the period of the data used for estimating the models.

Figures 4 and 5 compare actual daily loads with the daily forecasts generated by, first, the WS regression model alone, and second, the sum of the WS component model and a daily NWS component obtained by interpolating between the successive monthly forecasts of the Aoki-SSTS model, i.e., the white bars in Figure 3. For the month of July (Figure 4), *addition of the NWS component clearly moves the forecast in the right direction and reduces the severity of the underprediction.* For February (Figure 5), the adjustment provided by adding the NWS component is slight, but again it is in the right direction. In the early part of February the NWS component is positive, thereby reducing the underprediction error of the WS component. In the latter part of the month the NWS

Table 3. Reduction in variance of monthly average error term due to inclusion of NWS component estimated by Aoki-SSTS.

	Variance of Residuals from Equation (2)	Variance of Residuals from Equation (3)	Percent Reduction
Daily Sum	364994	196331	46%
Daily Max	1859	1083	42%
Daily Min	302	166	45%

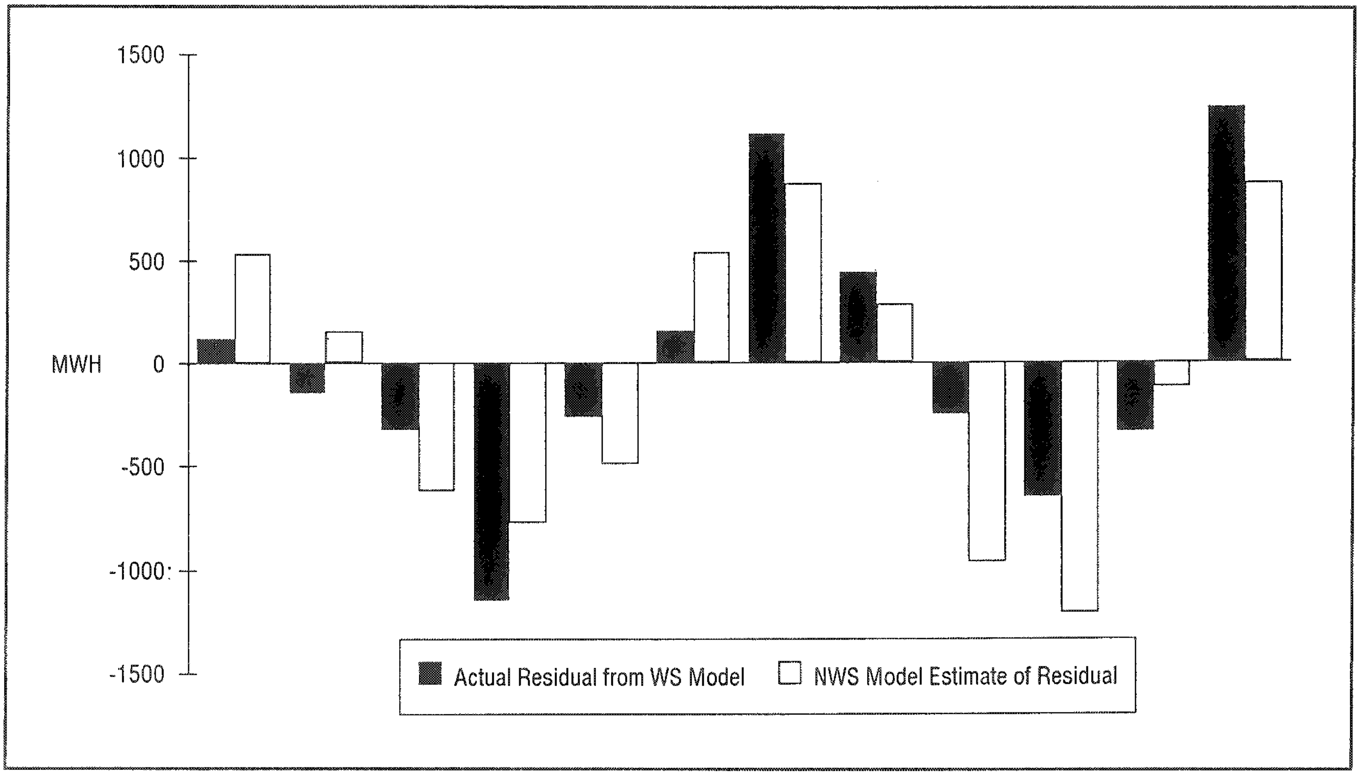

Figure 3. 1090 Monthly Average Residuals - Actual and NWS Model Estimates

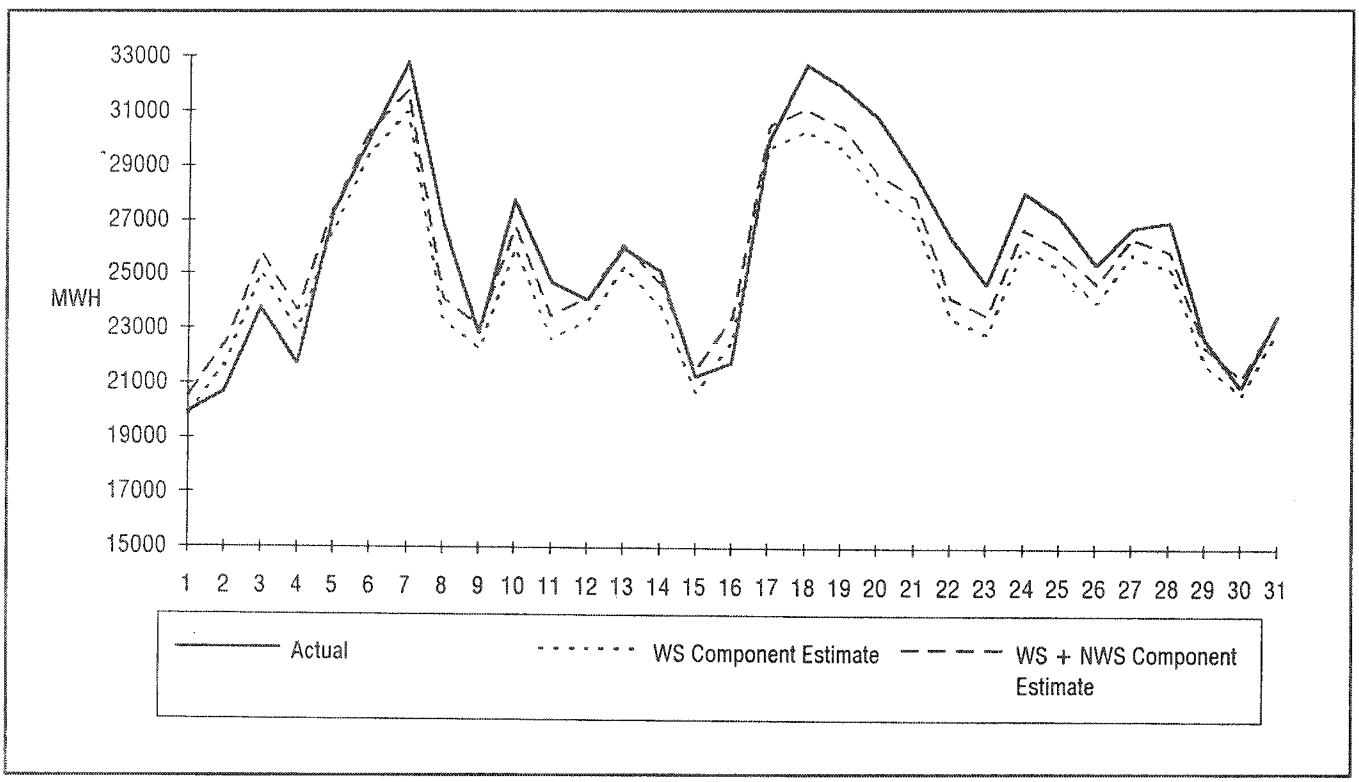

Figure 4. Total Daily Energy for July 1989

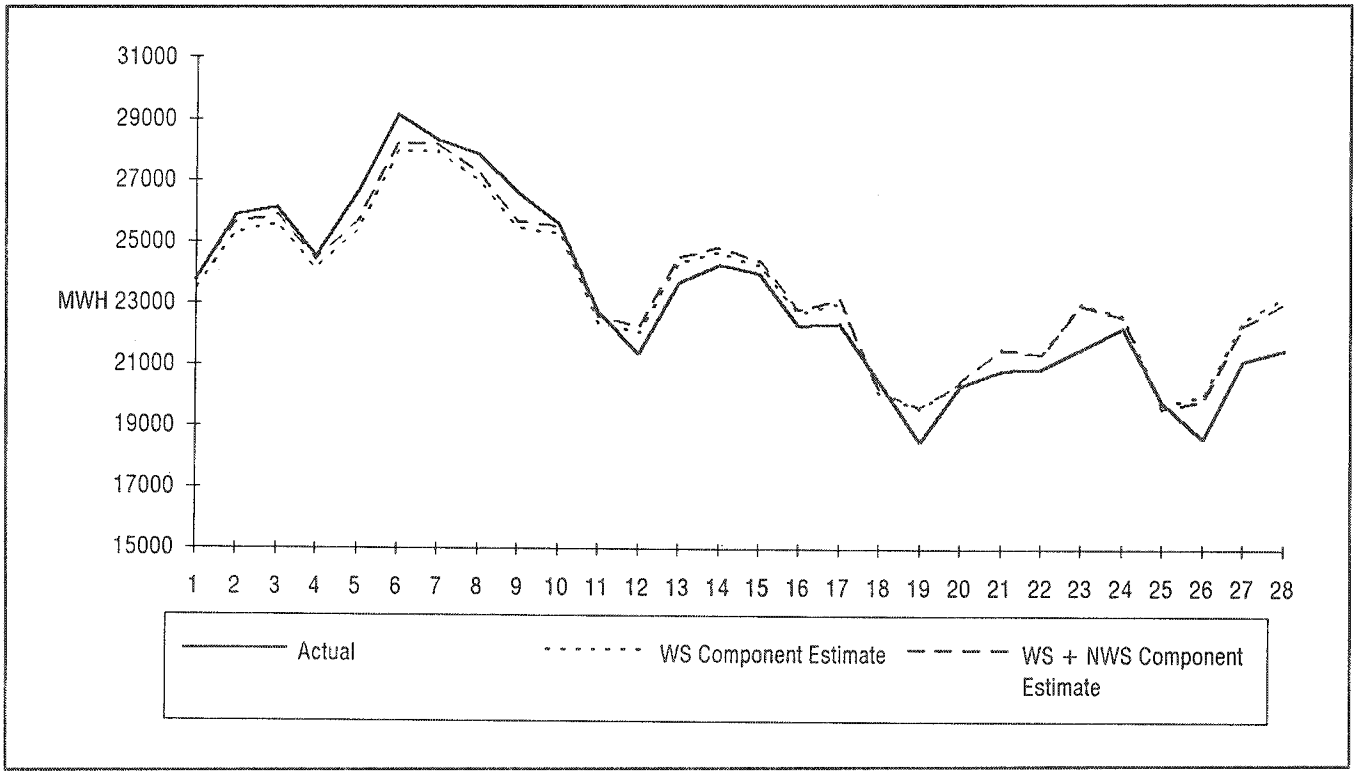

Figure 5. Total Daily Energy for February 1989

component is negative, thereby reducing the overprediction error of the WS component.

Conclusion

The principal contribution of this paper has been to propose and to initiate development of a new approach to allocating annual energy load forecasts to typical daily patterns and load profiles. The new approach takes into account both the variation explainable in terms of weather and other behavioral influences such as day length and holidays, and the residual variation that is harder to explain in causal terms.

At this stage of development the procedure shows promise, but also indicates clear directions for improvement. Even when the WS and NWS components are combined, there is systematic overprediction and underprediction. The NWS component does improve upon the unaided WS component, but the magnitude of the improvement is not yet large enough.

In summary, the combination of weather-based regression models with Aoki state space time series analysis appears to offer a promising approach for modeling and forecasting the annual pattern of energy demand variation. It is, therefore the intention of the authors to further develop and refine the approach, and to investigate its applicability to less aggregated measures of energy demand, and to more climatically diverse utility service areas.

Acknowledgments

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Endnotes

1. The method of generating 24-hour load profiles by estimating separate hourly models has been successfully used in other energy-weather studies. See, for example, the report by Regional Economic Research, Inc. (1990).

2. The regression technique used is ordinary least squares (OLS). Although it is well known that OLS is inefficient in the presence of autocorrelated residuals, the logic of the WS/NWS decomposition precludes the use of the standard corrections for autocorrelation.

More important in the present context are (a) the fact that OLS estimates of β are unbiased and consistent, and (b) having series of regression residuals that retain the autocorrelation structure to be captured in the NWS component model.

3. The concept of the "state space" originates in systems theory, a field of applied mathematics that specializes in modeling and estimating dynamical systems and which has recently been applied to time series analysis for its strength in forecasting. In building the SSTS model, the dimension of the state variable Z_{it} is crucial, for the states are the minimal sufficient statistics for the past history of the time series. The dimension of Z_{it} is determined by numerically finding the rank of a Hankel matrix, constructed from the three-by-three correlation matrices between past and future observations of the series making up the vestor ν_{it}. The device for finding this rank is the singular value decomposition, also used to estimate the matrices A and C in equations (3) and (4). After A and C are estimated, the matrix B in equation (4) is estimated by solving a matrix Riccati equation. For the full details of the procedure, see the paper by Aoki and Havenner (1991); for a richer theoretical exposition see Aoki's (1987) book.

4. See California Energy Commission (June 1991), particularly chapter 7; also Ignelzi and Way (1989) and Kristov (1991).

5. See Lunde (1980) pp. 64-75.

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