NAC for Linear and Change-Point Building Energy Models

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The Normalized Annual Consumption (NAC) index has proved useful in energy analysis using the PRISM model. This paper develops NAC with rigorous statistical error diagnostics for linear and four parameter change-point energy models. The models are applied to daily data from several case study commercial/institutional buildings, and the NAC estimates are analyzed and compared. The importance of goodness-of-fit for a model toward producing an accurate NAC estimate is examined. A statistic NSR, designed specifically for this task, is introduced.

Introduction

Procedures used to determine energy savings resulting from energy conserving retrofits generally utilize a model for the consumption, E_{pre}, of the building in the preretrofit configuration. E_{pre} is a function of one or more primary influencing parameters such as ambient temperature, and the model is typically developed using monitored preretrofit data when available.

Quite early during the development of the PRISM model, it was found to be valuable to determine a normalized value of the consumption (and subsequently the annual savings) corresponding to the consumption (and the savings) occurring during a typical weather year. Savings from a retrofit may be atypically low or high if the weather or other influencing parameters are abnormal during the period of measurement, and the building owner or others may be interested in the consumption and savings during a "normal" year. The consumption value during this "normal" year is defined as the "normalized annual consumption" (NAC) (Fels 1986). The PRISM NAC is defined for the three-parameter change-point model used in PRISM. However, the energy consumption affected by retrofits is modeled better by four-parameter change point or linear regression models (Kissock et al., 1992) in numerous commercial buildings where heating and cooling occur concurrently. A normalized measure of consumption for these models has not been previously defined.

This paper:

- Gives a definition for NAC that is independent of the building energy model used, so that NAC can be compared between models;
- Derives usable formulae for NAC for two-parameter linear and four-parameter change-point models;
- Develops a rigorous statistical error analysis for NAC with each model;
- Tests the error diagnostics on synthetic data;
- Computes NAC using these models on the same buildings and assesses the performance of each model in estimating NAC;
- Determines the importance of goodness-of-fit toward producing an accurate NAC estimate for daily data; and
- Introduces a statistic NSR helpful in identifying the quality of a model's fit and NAC estimate, relative to long-term weather data.

A General Definition of NAC

The normalized annual consumption (NAC) for a building is defined as the energy consumption value of the building during a "normal" year for the influencing parameters. While the term "normal" is somewhat vague, we take it to mean a year of (long-term) average conditions on the influencing parameters. For a year with normal conditions, let $\hat{E}_i$ denote the estimated energy consumption value on day i according to any given energy model. The sum of these estimated consumption values over the entire normal year is the building's NAC:

$$NAC = \Sigma \hat{E}_i \tag{1}$$

This general definition of NAC is independent of the model used to estimate the energy consumption, so NAC estimates calculated from this definition with different models are comparable. Equation (1) is valid for any energy model, but more practical and insightful equations can be derived from it. In particular, the familiar formula for PRISM NAC can be derived from this definition (Fels 1986), as well as NAC formulae for the linear and four parameter change-point models discussed in subsequent sections.

NAC for Linear Models

The variable-base degree-day model incorporated in PRISM is basically a linear model of energy consumption as a function of temperature except that it assumes that consumption is constant at a nonzero value to one side of a reference or base temperature. This model has a strong physical basis in many cases, e.g., residential electricity consumption has a non-weather dependent component due to normal household uses below a temperature, Tc, It then increases with temperature as cooling energy is required in addition to the other uses. Heating and cooling consumption in large buildings is often an approximately linear function of ambient temperature. This can be seen by examination of system and load equations for common air-side systems as given by Mitchell (1983), Knebel (1983) and others. It is also evident from examining monitored consumption data (Kissock et al. 1992) that consumption is primarily a function of temperature, with other parameters important in some cases. Kissock et al. examined both chilled water and steam data for 19 buildings--a total of 38 channels of thermal data--and found that a simple linear function of temperature provided a good fit for 15 of the 38 channels while a model linear in temperature and electricity consumption provided a good fit for an additional 7 channels.

We emphasize that these models are for commercial buildings where heating and cooling occur both concurrently and year-round. Due to climate and these facts, the linear models are valid for the year-round temperature range of the geographic region.

The Two Parameter Linear Model

Daily to monthly energy consumption data for a significant number of commercial/institutional buildings can be modeled adequately with a simple energy model E = a + bT, where E is the energy consumed and T the ambient temperature (Kissock et al. 1992). Our derivation of NAC for this model begins with the general definition of NAC.

For an average year of daily temperature data $\{T_i\}$, the estimated daily energy consumption value $\hat{E}_i$ for day i is defined by

$$\hat{E}_i = a + bT_i.$$

Thus

$$NAC = \Sigma\hat{E}_i = \Sigma(a+bT_i) = 365a + b\Sigma T_i$$

so

$$NAC = 365(a+b\langle T\rangle) \quad (2)$$

where <T> is the *long-term* average daily temperature for the region.

We pause briefly to observe that NAC is linear in <T>, rather than the average number of degree days as in PRISM. This is so because the linear relationship between temperature and energy consumption does not break down at a reference temperature within the year-round temperature range, due to the concurrent heating and cooling in commercial buildings of this type.

To obtain the standard error for NAC, s.e.(NAC), the basic properties of variance (var) and covariance (cov) are applied (see any introductory statistics book):

$$s.e.(NAC) = 365[var(a) + 2\langle T\rangle cov(a,b) + \langle T\rangle^2 var(b)]^{1/2} \quad (4)$$

Equations (2) and (3) are easy to use in practice because any standard software package can produce the values for var(a), var(b) and cov(a,b) when the model is being fit.

The parameter NAC has proved to be very reliable for the PRISM change-point model, in the sense of small percent standard error of NAC, or CV[NAC], even when the other model parameter estimates had a large degree of uncertainty (Fels 1986).

This stability of NAC will hold for the linear model as well. Consider the following intuitive argument. Regardless of the scatter of a data set (T_i, E_i), a linear fit to the data will always pass through its "centroid" (avgT, avgE), where avgT and avgE are the mean values of T and E for the given data set, and the estimate a + b*avgT = avgE of the true mean of E will be quite reliable for any data set with a reasonable temperature range. The level of certainty for a + bT will be high if T is in the middle of the data set, close to avgt. When <T> is close to avgt, as will be the case for most multi-month data sets, it is intuitively clear that NAC = 365(a + b <T>) will be a stable parameter (see Figure 1).

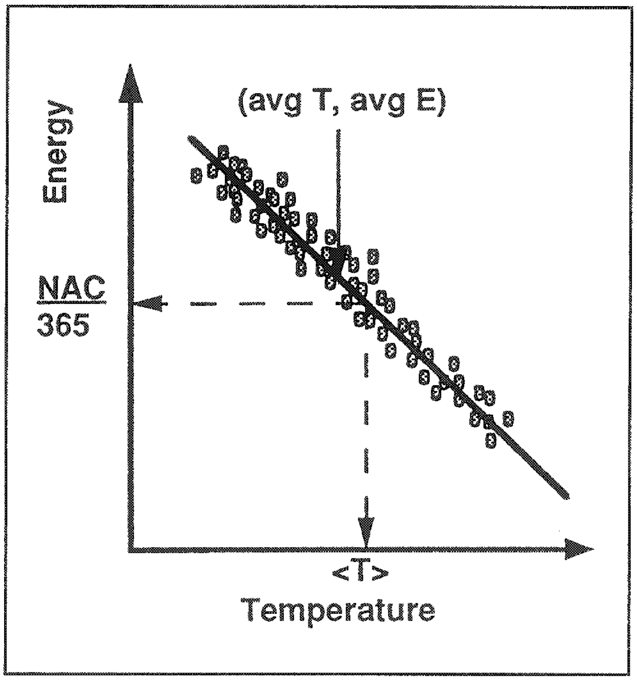

Figure 1. Stability of NAC. If <T> is close to AvgT, the NAC estimate will generally be more reliable than the estimates of parameters a and b.

In fact, it can be shown that for most reasonable data sets, and all physically meaningful models

$CV[NAC] < CV[b]$,

and

$CV[NAC] < CV[a]$

for cooling models (see Ruch 1991 for details).

The NAC estimate for the linear model is sensitive to the <T> value chosen. From Equation (2), we see that an error of Δ<T> in the long-term temperature average implies an error of ΔNAC = 365b*Δ<T>, which may be substantial if the slope b is large. The standard error of NAC will also be affected (see Equation (3)).

The number of years of long-term temperature data needed to compute <T> depends on weather fluctuations in the region, and a systematic study of this question would be worthwhile. The recommendation of Fels (1986) for the PRISM model is to use at least 10 years of data for computing the average number of degree days. However, because only <T> is needed for the linear model, a 30-year average (which is certainly sufficient) can be obtained from climatological records for the region.

Library Building Results

Description of the Building - Data

The case study is a heating model for a library in central Texas. Daily data for steam consumption and ambient temperature were collected between November 1990 and June 1991. All data collected were clean, and the majority of the year's heating season was included, so the data were considered acceptable for a heating model study.

A plot of the steam data (Figure 2) reveals no clear change-point temperature, which suggests a linear heating model H = a + BT.

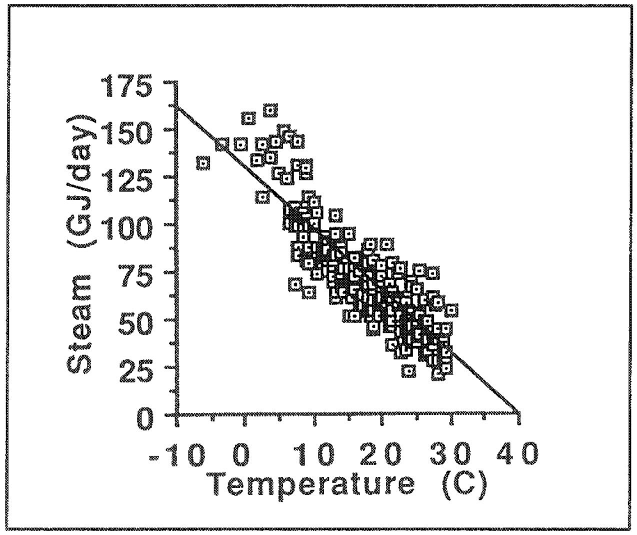

Figure 2. Library Data with Linear Fit

Stability of NAC and Sensitivity to <T>

The data from the building were fit with the two-parameter linear model, using eleven years of local National Weather Service data to compute <T>. The level of certainty for the NAC estimate is considered, and its sensitivity to <T> is discussed.

The linear model of steam use as a function of temperature for the building is

$$H = 129.25 - 3.376T \qquad (GJ/day) \tag{5}$$

where T is in degrees C. It should be noted that the data set includes nearly the entire temperature range of a normal year for the geographic region, so the model should be viable year-round.

The fit of the model is reasonable (R^2 = 0.77) and estimates of NAC and the slope b are very stable (Table 1). As discussed previously, CV(NAC) is lower than CV(b) or CV(a).

Table 1. Library Results for Linear Model

Parameter	Estimate/ std error	CV (Parameter)
a	129.3/2.4	1.85%
b	-3.4/0.1	3.62%
NAC	21939/345	1.62%

To measure the sensitivity of the NAC estimate to the long term average temperature (<T>), suppose the value of <T> were incorrect by AT = IC. This would change the estimate of NAC by ΔNAC = 1232 GJ, which is 5.6% of the NAC estimate. In addition, ANAC is approximately 3.5 times the estimated standard error for NAC, a considerable amount. Thus, it is important to use a reliable value for <T> if s. e. (NAC) is to be taken seriously.

The Nonlinear Four Parameter (FP) Model

The four parameter change-point model of energy consumption was introduced by Schrock and Claridge (1989) on physical and empirical grounds to explain electricity use of a grocery store. Ruch and Claridge (1991) provided a statistical justification and error diagnostics for the model and suggested that it would be applicable to data from a large number of commercial buildings. Kissock et al. (1992) found it to be the model of choice for 14 of 38 channels of thermal data analyzed.

The four-parameter change-point energy model is

$$E = a + b_2(T-t)^+ - b_1(t-T)^+ \quad (6)$$

where T is the average daily temperature, t is the change-point temperature, b_1 is the low temperature slope and b_2 is the high temperature slope (see Figure 3.) To derive the NAC for this model, we begin with the definition of NAC. For an average year of temperature data $\{T_i\}$,

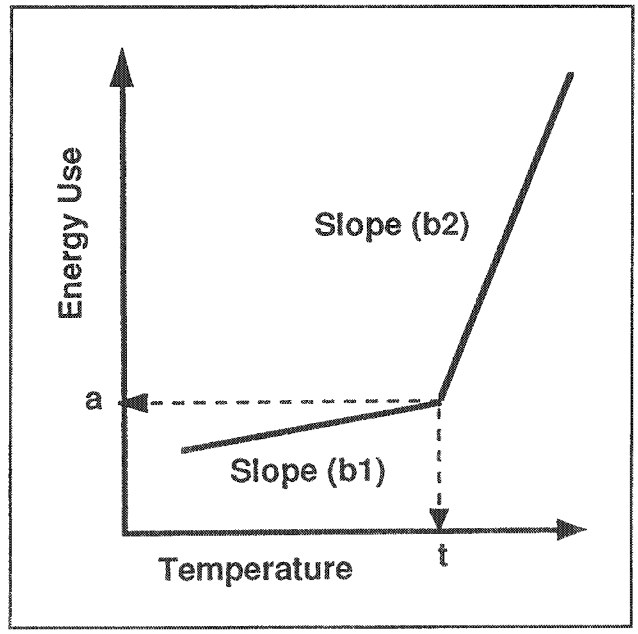

Figure 3. Four-Parameter Model. The change point coordinates are (t,a), the lower temperature region slope is b_1, and the high temperature region slope is b_2.

$$NAC = \Sigma \hat{E}_i = 365a + b_2\Sigma(T_i-t)^+ - b_1\Sigma(t-T_i)^+$$

so (7)

$$NAC = 365a + b_2 DD_+(t) - b_1 DD_-(t)$$

where $DD_-(t)$ [$DD_+(t)$] is the average number of degree days below [above] the reference temperature t in an average year.

Note that dropping the $DD_-(t)$ [$DD_+(t)$] term will yield the cooling [heating] formula for NAC used in the 3 parameter PRISM model. Like PRISM NAC, the FP model's NAC is linear in the average number of degree days, but both $DD_-(t)$ and $DD_+(t)$ are required since the temperature slopes are non zero on both sides of the change-point temperature.

Since the FP model is nonlinear, the simple approach used in calculating s.e.(NAC) for the linear models will not work here. Instead, the Equation (5) is reparameterized to make NAC an explicit parameter in this energy model. A likelihood-based standard error and confidence intervals for NAC can then be calculated. A description of this process is given in the next section.

Calculating the Standard Error and Confidence Intervals for NAC

To reparameterize Equation (5), Equation (6) is solved for the parameter a; this expression is then substituted into (5) yielding

$$E = \frac{NAC}{365} + b_2 UPT - b_1 DNT \qquad (8)$$

where

$$UPT = (T-t)^+ - \frac{DD_+(t)}{365} \text{ and } DNT = (t-T)^+ - \frac{DD_-(t)}{365}.$$

With NAC now an explicit parameter in the energy model, a confidence interval for NAC is defined as follows:

For a set of parameter estimates b_1, b_2, NAC, and t, let RSS(b_1, b_2, NAC, t) denote the residual sum of squares for Equation (5), and $\hat{RSS}$ denote RSS($\hat{b}_1$, $\hat{b}_2$, $\hat{NAC}$, $\hat{t}$), where $\hat{b}_1$, $\hat{b}_2$, $\hat{NAC}$, $\hat{t}$ are the parameter estimates giving the best least-squares fit to the data. A point x lies in the confidence interval for NAC if, and only if, there exist b_1, b_2 and t such that

$$RSS(b_1, b_2, x, t) \leq R\hat{S}S(1 + \frac{1}{m-4} F^e_{1;m-4}) \qquad (9)$$

where e is the significance level of the confidence interval, m is the number of data points and $F^e_{1;m-4}$ is the F statistic with 1 and m-4 degrees of freedom.

We define the standard error (with significance level e) for a parameter to be half the length of the confidence interval. The statistical theory behind the confidence intervals is discussed in Beale (1960) and Hinkley (1969). The likelihood-based approach used here is recommended by these authors. The algorithm used to compute the endpoints of the confidence interval for NAC does so by finding, approximately, the extreme values of NAC (MAXNAC and MINNAC) for which inequality (8) holds for some values of b_1, b_2 and t. This is done by searching along the t-axis, to the right and left of $\hat{t}$ in increments of 0.1, for values of t such that inequality (8) is satisfied. For each such value of t, the corresponding extreme values of NAC satisfying (8) are found, and the "global" values MAXNAC and MINNAC updated if necessary. When the search for acceptable values of t is exhausted, the final values of MAXNAC and MINNAC are recorded as the endpoints of the confidence interval for NAC. Confidence intervals for the other parameters are found similarly. A program implementing this algorithm has been written and used for the computations of this paper. See Ruch and Weber (1990) for complete details.

Testing Error Diagnostics on Synthetic Data

The standard error for NAC should be tested on synthetic data for two reasons. The parameters of Equation (5) have been shown to be asymptotically normal (Feder 1975), so the error diagnostics output by the program should be very precise for large samples. The program's output must be tested, first of all, to establish this as fact. Secondly, the error diagnostics must be tested on smaller samples to ascertain the sample size necessary for reliable standard errors.

Three numerical tests were carried out to validate the algorithm's NAC estimate and error diagnostics for different data sample sizes. In each test, 200 data sets were generated using random normal deviates on Equation (5) with parameter values $b_1 = 40$, $b_2 = 200$, a = 8000, t = 15 and variance or $\sigma^2 = 400^2$. The data sets for these tests had sample sizes of 365, 120, and 12, respectively, with temperature values $\{T_i\}$ chosen randomly from a 10 year data set of average daily temperatures in the College Station, Texas area. For all tests, a four-parameter fit and accompanying parameter confidence regions at significance levels of 0.32, 0.05 and 0.01 were constructed for each data set. The mean value of the NAC estimates and the number of confidence intervals containing the true value of NAC were tabulated for each test (Table 2).

The results for the tests indicate that the approximate confidence intervals computed according to our algorithm are very accurate, because they contain the actual NAC value at essentially the same percentage rate expected. The average NAC estimates are quite accurate for the large and moderate sample sizes, but not so good for the small one. If daily data are used for an energy model, several months of data should be used to ensure quality performance by the algorithm.

Table 2. Numerical Test Results. The entries in the significance level columns give the percentage of the 200 approximate confidence intervals containing the true value of NAC at the indicated significance level. The expected values for the column's entries are 68, 95, and 99, respectively.

Test (sample size)	Significance Level			True NAC (kWh)	Ave $\hat{NAC}$ (kWh)	% Error
	0.32	0.05	0.01			
No. 1 (365)	68	95	99.5	3212000	3212004	0.00012%
No. 2 (120)	68	94.5	99	3212000	3211059	-0.011%
No. 3 (12)	66	94	98	3212000	3203556	-0.26%

Comparison of NAC Estimates by the Linear, Prism and Change-Point Models

The three models were applied to sets of daily data from three commercial buildings in central Texas. The goodness-of-fit and NAC estimates are compared, with the goal of determining the importance of goodness-of-fit to the accuracy and level of certainty of a NAC estimate.

Description of the Building Data

For all three buildings, the daily energy consumption was calculated from whole-building hourly data. See Claridge et al. (1991) for details of the LoanSTAR Project data collection techniques.

The daily chilled water (CW) data for a drama building were collected from October 1990 through April 1991, with data from 2 of 190 days missing, prior to a retrofit to the building in May 1991. A NAC estimate for the cooling model is particularly interesting because it is necessary for determining retrofit savings. The estimate is also somewhat difficult because data from the high temperature region were largely unavailable.

The daily steam data for a heating model of a chemistry building were collected from October 1990 through June 1991, with data from 5 of 256 days missing. All remaining data were clean, with the exception of one week in February when construction caused abnormal energy use. The data from this week were excluded when fitting the model.

The grocery store electricity data for a cooling model covered 324 days in the period July 1989 - June 1990. Complete details are available in Ruch and Claridge (1990.)

These buildings were chosen for analysis because they all displayed change-point behavior in varying degrees. The chemistry building clearly deserves treatment by the FP model, while a plot of the drama building data seems nearly linear, and the grocery store appears accessible to a PRISM fit (see Figures 4,5,6.)

Application of the Models

All three models were fit to each building, and NAC estimated using an eleven year set of daily temperatures from the region. The goodness-of-fit and NAC statistics are detailed in Table 3.

In all three buildings, the four parameter (FP) model's fit of the data was (as expected) the best; significantly so for the chemistry building. In the grocery store case, all three models gave similar NAC estimates, with small and reliable CV[NAC] estimates. Even though the linear cooling model is incorrect for this data (Figure 4), the large proportion of data points above the reference temperature, where the cooling load was also the greatest, forced a linear fit that was acceptable in the high temperature cooling region, and consequently produced a reasonable NAC estimate.

Let us emphasize this point: if the model cannot fit the entire data set well, a good NAC estimate can only be expected if the majority of the data lies in the high

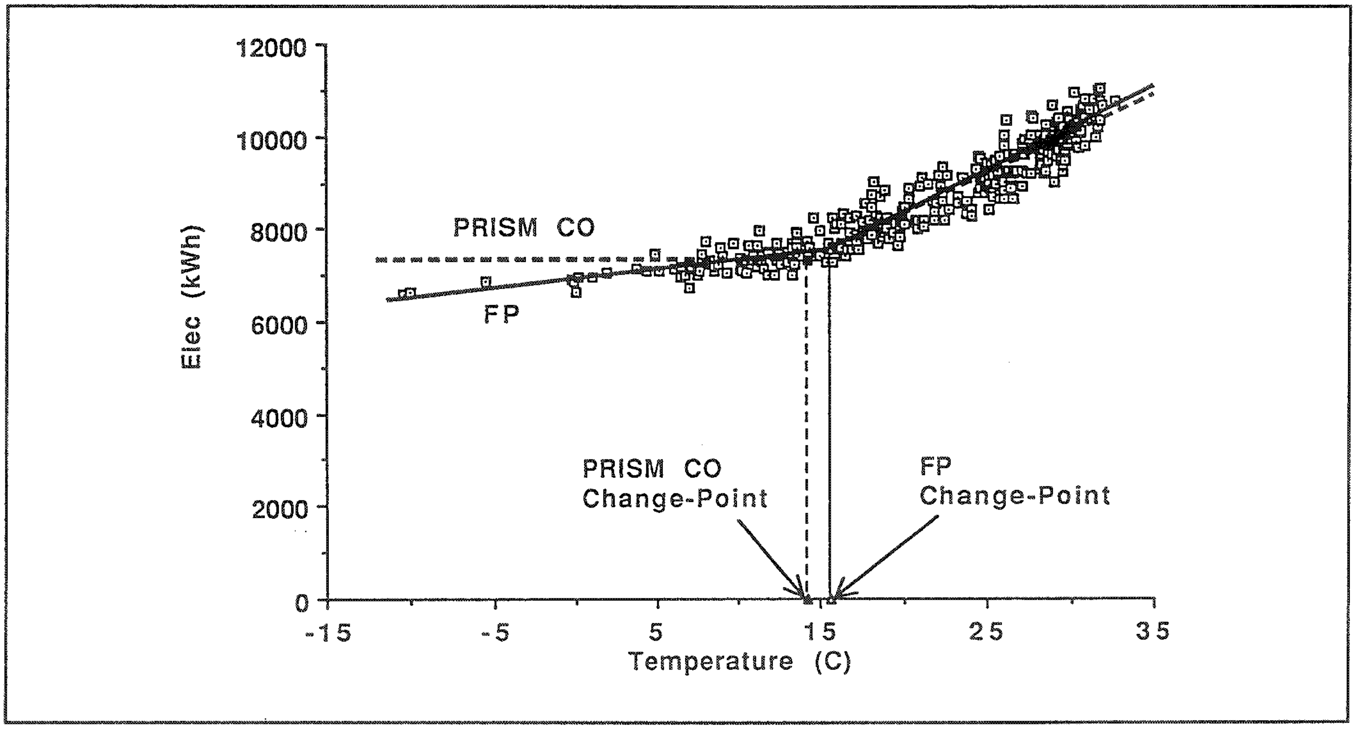

Figure 4. Grocery Store Data with PRISM CO and FP Model Fits

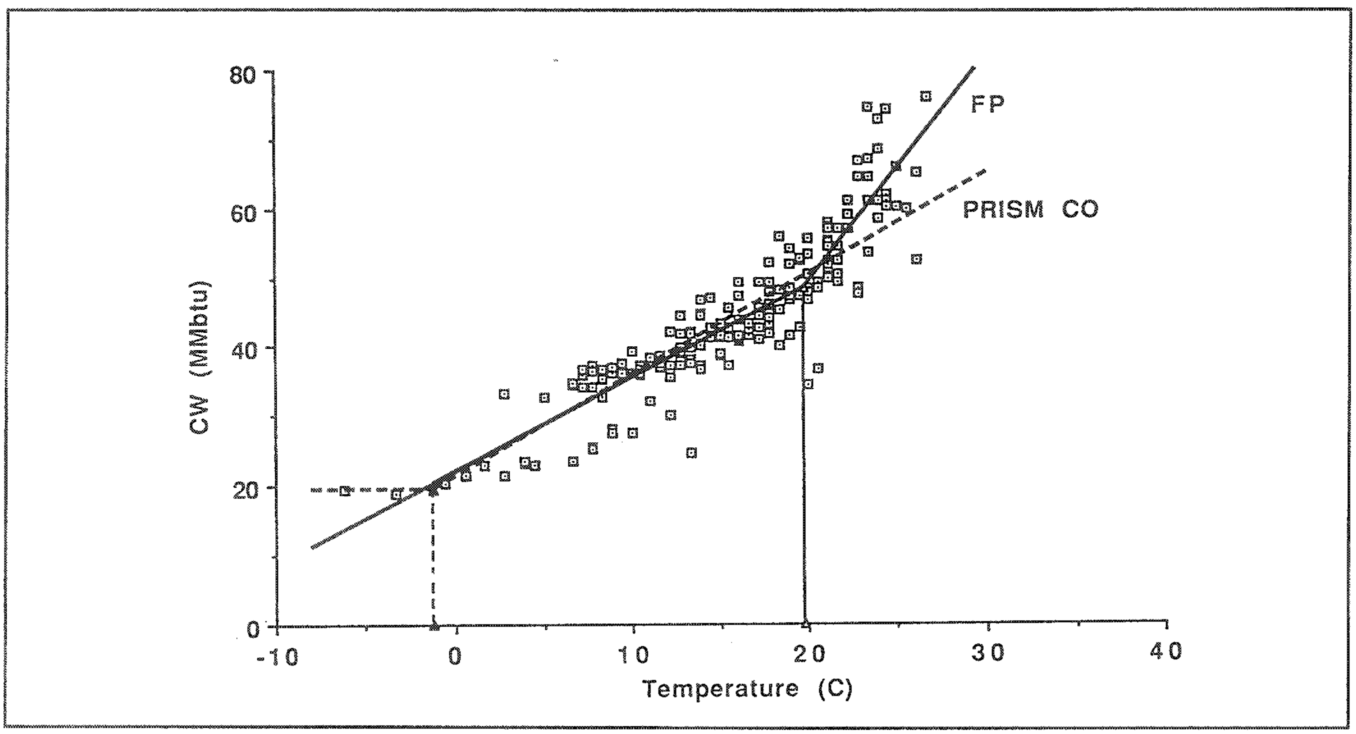

Figure 5. The Drama Building Data with PRISM CO and FP Model Fits. The PRISM CO fit above 21°C is too low.

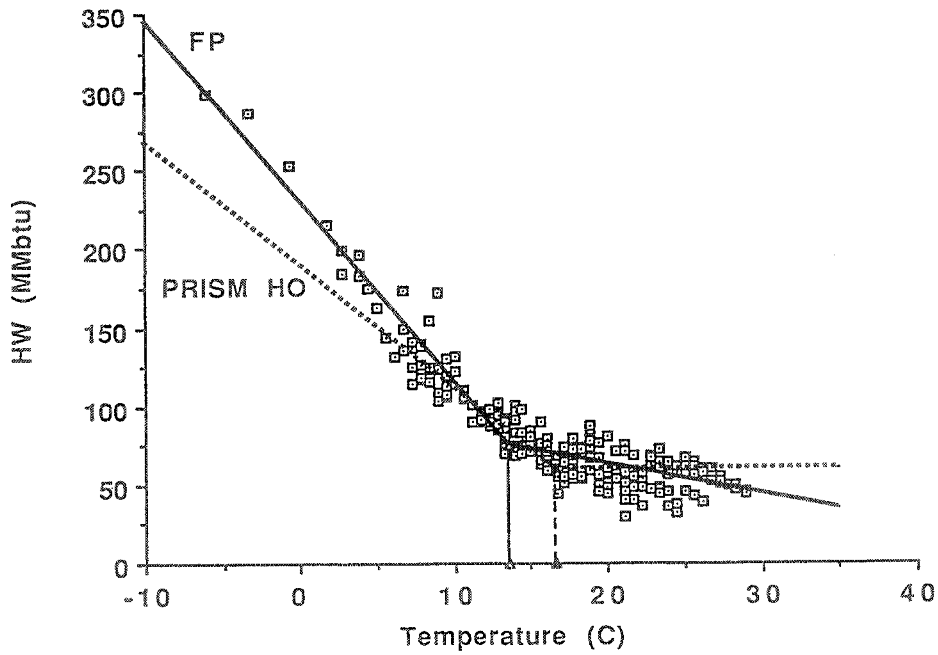

Figure 6. The Chemistry Building with FP and PRISM HO Model Fits. The PRISM HO fit is too high above 24°C. Forty percent of the days in an average year will lie above 24°C.

consumption region. For this reason the FP model has an advantage over the simpler models, in that its superior goodness-of-fit will make the structure of the available data set less important for a reliable NAC estimate.

Turning to the drama building, what worked for a linear fit to the grocery store data works against it with the drama building, since data for this cooling model were only available from November 1990 through April 1991. This structure of the data set resulted in a PRISM fit, that appears to underestimate consumption for temperatures above 21°C (See Figure 5) where most of the consumption takes place on an annual basis, and consequently gives a NAC estimate 9.8% less than that of the FP model, while the PRISM CV[NAC] is only 1.4%. The linear NAC estimate is 7.7% lower than the FP model's estimate for similar reasons. Judging from the FP model's superior goodness-of-fit and the above analysis of the data set, the estimates of NAC and CV[NAC] for the drama building from the linear and PRISM models must be considered inaccurate and unreliable.

For the heating model of the chemistry building, the data set included most of the annual heating period (Figure 6). While the superior goodness-of-fit given by the FP clearly indicates that its NAC estimate is to be preferred, the results produced by the linear and PRISM models are reasonable: The PRISM HO NAC estimate is about two PRISM standard errors above the FP NAC estimate (about 5% of the FP NAC estimate). A close look at Figure 6 for temperatures above 24°C shows that the PRISM HO is over-predicting in this region. Although consumption is fairly low per day, about 40% of the days in an average year fall in this region. This appears to be the reason for PRISM HO's overestimate of NAC.

Identifying a Model's Lack of Fit, Normalized to Long-Term Weather: The NSR Index

Traditional goodness-of-fit measures such as R^2 may not emphasize temperature regions of long-term importance, as seen in the previous section for the chemistry and drama buildings. For this reason, we now define the Normalized Systematic Residuals Index (NSR), which gives a percentage measure of a model's residuals over temperature regions defined by long-term temperature distribution. A large value of NSR indicates that a model is consistently over or underestimating the data over a significant long-term temperature region. NSR can be compared across both models and buildings.

For a model being fit to data $\{T_i, E_i\}$, let L be the subset of the data lying in the lower half of the long-term temperature distribution line (i. e., for T_i less than the

Table 3. Goodness-of-fit and NAC statistics for Model Comparison. High values of NSR indicate a systematic lack of fit over a region of the data, which may result in a poor NAC estimate and unreliable CV(NAC). The NAC estimates for the grocery store is in kWh, while the NAC for the chemistry and drama buildings are given in MMbtu.

	Chemistry	Grocery	Drama
Linear			
R^2	0.73	0.85	0.80
NSR	10.5%	0.96%	6.3%
NAC	24565	3148078	19133
s.e. (NAC)	511	9197	174
CV (NAC)	2.1%	0.3%	0.9%
Prism			
R^2	0.62	0.87	0.57
NSR	9.4%	0.1%	9.4%
NAC	27733	3130871	18690
s.e. (NAC)	590	8598	261
CV (NAC)	2.1%	0.3%	1.4%
Four-Parameter			
R^2	0.92	0.92	0.84
NSR	1.0%	0.3%	1.5%
NAC	26402	3140272	20728
s.e. (NAC)	305	6894	324
CV (NAC)	1.2%	0.2%	1.6%

long-term median temperature.) Similarly, let U denote the upper portion of the data. We also need an "average" energy value for the data set, weighted according to the long-term temperature distribution:

$$WAVG(E) = 1/2\,[AVG_L(E_i) + AVG_U(E_i)]$$

where $AVG_L(E_i)$ $[AVG_U(E_i)]$ denotes the average energy value over the lower [upper] portion of the data.

The NSR Index is then defined to be:

$$\max\left\{ |AVG_L(Resid)|,\ |AVG_U(Resid)| \right\} / WAVG(E)$$

If the model's fit is good in each of the long-term temperature regions, zero is the expected value of both $AVG_U(Resid)$ the average residual of the model over the upper half of the long-term temperature distribution, and $AVG_L(Resid)$. Thus a small value for NSR indicates a good fit to the data and, hopefully, a good NAC estimate. On the other hand, a large NSR value says that the model's residuals are large as a percentage of the weighted average energy consumption.

NSR is defined in terms of residuals rather than square errors in order to detect systematic over or underestimating by the model. The low values of NSR (less than 2%) in Table 3 are consistent with the NAC estimates considered good in the previous section. This is not always the case with R^2 or CV[NAC] values (see especially the linear model of the drama building.)

These results can be summarized in a CV[NAC] against NSR plot (Figure 7) in the spirit of CV[NAC] against R^2 plots used by Reynolds (1990) in a PRISM study. Building models with low CV[NAC] and NSR values have good NAC estimates. From the previous discussion, it appears that models with NSR $\leq 2\%$ and CV[NAC] $\leq 2\%$ are reasonable for large commercial buildings with daily data. However, more work should be done testing the value of NSR on other buildings.

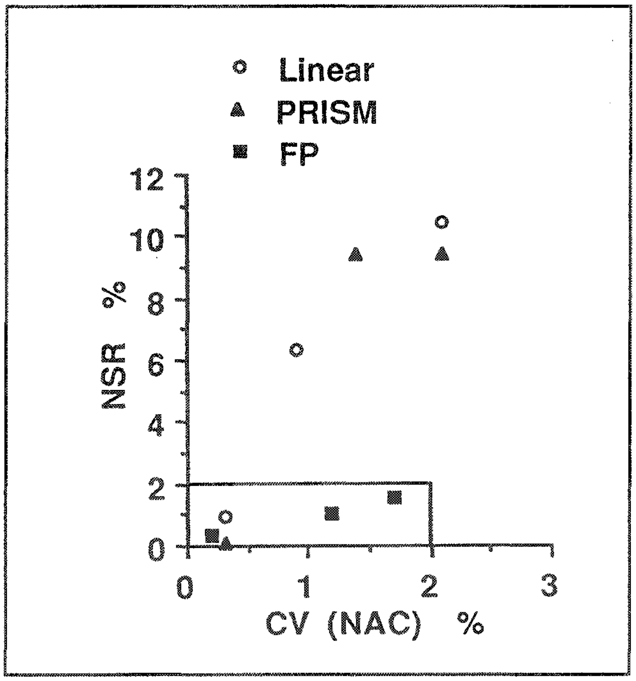

Figure 7. NSR vs. CV[NAC] Summary. Buildings with low NSR and CV[NAC] values have reliable NAC estimates. Linear, PRISM, and FP models of the three buildings are shown.

The NSR index could easily be defined by splitting the data into three or more portions rather than two. While this was not necessary to detect the systematic residuals in the cases mentioned above, it merits further study.

Conclusions

The availability of NAC with rigorous error diagnostics for several energy models allows more choices for fitting a physically meaningful model to data and determining a reliable NAC estimate.

For the case studies, it seems that good fits to the data in normal year high consumption regions are essential for obtaining a good NAC estimate with a reliable standard error. The term "high consumption region" encompasses two important cases: regions where the daily energy use is generally high, and regions that include a large percentage of the days in a normal year.

The FP model allows for NAC estimates more accurate than those given by linear and PRISM models for some buildings. The fundamental reason for this is the greater goodness-of-fit offered by the FP model. Tlis makes the FP model less dependent on the structure of the data set for reliable NAC estimates. In particular, the FP model may be especially useful when data from high consumption regions is scarce.

The NSR index defined in this paper identifies a model's systematic lack of fit in temperature regions of long-term importance. For the case studies exandned, the NSR index appears promising as a tool for assessing the quality of a NAC estimate.

Since different NAC estimates were compared for only three buildings, NAC comparisons and the testing of NSR on more buildings should be undertaken.

Acknowledgments

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