Uncertainty Analysis in Estimating Building Energy Retrofit Savings in the LoanSTAR Program

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A crucial requirement to promote and sustain energy conservation measures in buildings is the ability to perform careful and reliable appraisals of exactly how much energy has been saved. This paper briefly describes the general issues involved and then focuses on commercial buildings where retrofits are made in order to reduce energy consumption in hot water, chilled water and in electricity consumed by air-handler and chilled water pumps. How retrofit energy savings in the framework of the Texas LoanSTAR project are estimated is described, and the need to assess the accuracy in these estimates themselves is highlighted. The various sources of uncertainties in the retrofit savings estimates are itemized and discussed. We point out the need to consider the strong serial correlations present in daily (and other types of time series) data and how this impacts the model identification process as well as the determination of uncertainty. Subsequently, equations to compute the prediction uncertainty associated with the use of a regression model involving serially correlated data streams and with error in the measurement data are presented. Use of these equations is illustrated by means of a case study involving one large commercial LoanSTAR building. Attention has been drawn to areas in which additional work is needed in order to improve the estimates of retrofit savings.

Introduction

Several large-scale building energy conservation programs in the United States have been initiated by electric utilities (Schuster and Tomich 1985), by state agencies (Claridge et al. 1991), and by federal agencies (Norford et al. 1986) to demonstrate that substantial reduction in energy use in residential and commercial buildings is both technically and economically feasible. These programs are generally based on one or several of the following: (i) retrofits to HVAC equipment, and replacement of equipment and lights, (ii) changes in building operation and schedule (for example, thermostat night set-back or lowering lights at night), (iii) modifications to the building shell, and (iv) continuous monitoring of energy use and analysis and control thereof (Haberl and Vajda 1988). A crucial requirement in being able to promote and sustain such energy conservation measures is the ability to perform careful and reliable appraisals of exactly how much energy has been saved. The accuracy with which the energy savings can or have been estimated is obviously a closely related and major issue, and is the focus of this paper.

Despite a number of major advances over the last two decades in analysis methodology and tools development relating to assessment of energy conservation in buildings (MacDonald and Wasserman 1988), additional refinements and extensions are still needed in order for these techniques to reach a level of maturity satisfactory and robust enough for use by the professional building community. In this paper, we shall limit ourselves to the issue of energy savings in commercial buildings, digressing to residential buildings only when relevant. Moreover, we shall also confine ourselves to instances when energy savings are realized as a result of retrofits to equipment, i.e., case (i) above. The present study has been performed in the framework of the Texas LoanSTAR program (Claridge et al. 1991).

This paper is divided into four sections. The first section describes the methodology for estimating retrofit savings in the Texas LoanSTAR program; the second section discusses the sources of errors generally present in the statistical measurement of savings; the third presents and discusses equations for determining the uncertainty of the estimated energy savings and the fourth section illustrates the approach with actual data from a LoanSTAR building.

Estimation of Retrofit Savings

One way of estimating retrofit savings is to directly compare the unadjusted pre-retrofit energy use to the post-retrofit energy use. Though this method may yield a first-order evaluation, it has generally been found to be too simplistic because the effects of the retrofit on energy use may be largely or entirely masked by changes between the pre- and the post-retrofit periods of certain important parameters influencing energy use (the most important often being the climatic variables) (Greely et al. 1990). Consequently, in order to incorporate the effects of such changes into the energy savings calculation, a theoretical model for building energy use needs to be developed. There are currently three types of modeling approaches:

i) Regression model approach (Fels 1986; Claridge et al. 1990; Kissock et al. 1992a, b);

ii) Calibrated model approach using existing hourly building simulation codes, (Hsieh et al. 1988; Bronson et al. 1992); and

iii) Simplified model approach (Rabl 1988; Subbarao 1988; Reddy 1989; Katipamula and Claridge 1992).

Whenever appropriate, model development using the regression approach is used because it is generally the least demanding in effort and user-expertise, yields adequate results and permits uncertainty associated with savings to be quantified using accepted statistical procedures. The calibrated simulation model approach is more tedious and requires knowledge of how the mechanical systems of the building are operated and a certain proficiency in using the particular building energy simulation code. It is typically resorted to only when the quality or length of the data period is not adequate to enable proper regression model identification. The simplified model approach (developed until recently essentially for residential building energy use) falls between the two approaches both in the level of user-expertise and length of data period.

A unique feature of the Texas LoanSTAR program, which is a program funding energy conservation retrofits in state, county and municipal government buildings and schools in Texas, is that data acquisition equipment to monitor building energy use be installed for a suitable period before the retrofits are carried out and remain in the building, possibly throughout the retrofit life (Claridge et al. 1991). Consequently, estimates of the retrofit energy savings can be based on the regression model approach. There are, however, buildings for which, due to a variety of reasons, the pre-retrofit data is either too short or even entirely unreliable. Only in such instances are the calibrated and simplified model approaches considered for use in the LoanSTAR program (Bronson et al. 1992; Katipamula and Claridge 1992). These have yet to reach a stage of maturity in methodology development where they can be used routinely with confidence. Consequently the rest of the paper pertains exclusively to the regression model approach.

As of October 1991, energy savings in eight LoanSTAR buildings are being reported (Kissock et al. 1992a). This number has increased to 14 buildings as of March 1992. These buildings are all located on university campuses and vary in size from 49,000 ft^2 to 484,000 ft^2 and house classrooms, offices, laboratories, computer facilities, auditoriums, workshops and a major campus library. All the buildings are provided with electricity, chilled water, and steam (or hot water) from campus utility plants that are separate from the buildings. The primary retrofit in all these buildings was the conversion of constant volume air handling units to variable air volume air handling units. The resulting energy savings in whole building chilled water use, whole building hot water use and electricity use of air handlers and chilled water pumps are individually estimated and reported. Thus, in general, a minimum of three regression models need to be developed for each building.

The methodology currently used to report retrofit savings in LoanSTAR buildings basically involves the following steps (Kissock et al. 1992a, b):

1) **Identification of the pre-retrofit, construction and post retrofit periods.** This is done both from log books and inspection of the hourly time series plots of air handler electricity use. Changes in consumption patterns are very distinct during these three stages and consequently there is little ambiguity at this stage.

2) **Preliminary data handling.** The entire data set from each building is composed of hourly averaged or summed observations of chilled water energy use, hot water energy use, whole-building electricity, air handler electricity use, chilled/hot water pump electricity use, and climatic variables (ambient dry bulb temperature, relative humidity, wind speed and global horizontal solar radiation). Each of these channels is screened and converted into daily averaged data (Lopez and Haberl 1992). This is the time scale presumed in all subsequent steps.

3) **Regression model identification.** Daily data from the pre-retrofit periods are used to develop regression models for daily chilled water, hot water, and air-handler electricity use. The single most important predictor variable for chilled water and hot water energy use is the ambient dry bulb temperature. Electricity consumed by lights and appliances, which is deduced from whole-building use and from air handler and chilled water pump use, is a secondary influential predictor variable because it is representative of the internal building loads. The importance of other variables is uncertain, and an

on-going effort in the LoanSTAR analysis group is to refine current model identification methodology so that the influence of additional variables can be rationally and systematically ascertained and included in the model if deemed necessary.

4) **Predicting energy use.** The regression model (identified using daily data) is used to predict daily energy consumption of the non-retrofitted building under building operation and weather conditions corresponding to each day of the post-retrofit period. Because the building has already undergone retrofits, the use of a model is unavoidable and leads to model prediction uncertainties which subsequently impact retrofit savings estimates.

5) **Estimation of savings.** Finally, the savings over a certain number of post-retrofit days are estimated by subtracting the daily measured energy consumption from the daily energy consumption predicted by the pre-retrofit model and summing the daily savings.

The entire procedure for computing total savings of either chilled water, hot water or electricity can be summarized by:

$$\sum_{j=1}^{m} E_{Save,j} = \sum_{j=1}^{m} \hat{E}_{Pred,j} - \sum_{j=1}^{m} E_{Meas,j} \tag{1a}$$

or

$$E_{Save,Tot} = \hat{E}_{Pred,Tot} - E_{Meas,Tot} \tag{1b}$$

where

- j = subscript representing a particular day over the post-retrofit period
- m = number of post-retrofit days over which savings are estimated
- $E_{Save,j}$ = energy savings over day j,
- $\hat{E}_{Pred,j}$ = model predicted pre-retrofit daily energy use
- $E_{Meas,j}$ = measured post-retrofit daily energy use
- Tot = subscript for "total" over the entire m days of the post-retrofit period.

It is clear from the above discussion that the regression model identification phase is crucial in the entire retrofit savings process. Currently, most regression models are linear, and of the first-order. This was dictated by (i) preliminary and previous experience supported by heat transfer and thermodynamic principles that energy flows in buildings could be well represented by linear first order functional forms (Fels 1986; Rabl 1988; Subbarao 1988), and (ii) a desire to keep the statistical identification simple (proper non-linear regression requires a much higher level of expertise). This aspect of model development is also being currently assessed and refined.

It is well known that energy use in buildings often exhibits change-point or segmented linear behavior with ambient temperature (Fels 1986; Ruch and Claridge 1991; Kissock et al. 1992). In residential buildings, where concurrent heating and cooling is not required, the presence of a change point is obvious (Fels 1986). In commercial buildings, however, interior zones may require cooling while the exterior zones may call for heating. This, coupled with the fact that HVAC deck temperatures are controlled non-linearly with ambient temperature, often results in change-point behavior. Consequently, the regression models can be sub-divided into three groups:

i) Mean or one-parameter models (for example, air handler electricity use for constant volume dual duct systems in buildings is reasonably independent of weather and a mean daily value has been found to be adequate for most commercial buildings (Kissock et al. 1992a),

ii) Simple or multiple linear regression models (MacDonald and Wasserman 1988), and

iii) Change-point or segmented linear regression models, which can be further sub-divided into:

   - Three-parameter or PRISM models (Fels 1986), and

   - Four-parameter or 4-P models (Ruch and Claridge 1991).

Identification of model type (i) is relatively straightforward and can be done in standard packages (for example, we use SAS, 1989 and STATGRAPHICS, 1991). Though linear segmented models are special cases of a much larger set of models, called spline functions (Pindyck and Rubenfeld 1981), these commercial packages do not, however, allow segmented linear regression modeling to be investigated in a framework convenient enough for building energy analysis. This is because the change point needs to be known and specified in order to use classical spline regression. Because this is not known a priori, for buildings (in fact, this is one of the parameters being identified by regression), these commercial packages are inadequate. Another deficiency in these packages is the lack of proper error diagnostics for spline regression models. Consequently, specially written computer programs, like PRISM (Fels 1986) or in-house programs, like 4-P (Ruch and Claridge 1991), are used. In their current development, these programs suffer from the drawback that models with only one regressor variable can be evaluated. This deficiency is due to a lack of properly accepted statistical formalism and methodology of how to deal with change point behavior when more than one regression variable is present. However, the fact that the ambient temperature as the sole regressor variable is very often adequate to model energy use in many buildings has lessened the urgency to overcome this deficiency. The computational algorithm on which the 4-P model is based involves a search method where the residual sums of squares over each of the two segments are computed separately for each incremental variation in the change point temperature. These two values of the sum of squares are then added together. The particular value which minimizes this sum is said to correspond to the sought-after change-point temperature (Ruch and Claridge 1991). Extensions and improvements in the 4-P model are also being currently studied by the LoanSTAR analysis group.

The entire approach of model identification involves several important issues which need to be enumerated and discussed individually if one wishes to guard against misuse and drawing of simplistic conclusions. Also, model identification has direct bearing on determining the uncertainty of retrofit savings because the same issues equally affect the nature and magnitude of errors.

Sources of Uncertainty in Regression Models

The uncertainty in savings can be attributed to **measurement errors** (both in the independent and dependent variables) and to **errors in the regression model**. The former are relatively well known to engineers and the methodology of estimating their effect is adequately covered in classical engineering textbooks, for example, Schenck (1968), and Bendat and Piersol (1986). Errors in regression models, on the other hand, are more complex and arise from several sources. They can be classified into four categories:

(a) **Model prediction errors** which arise due to the fact that a model is never "perfect." Invariably a certain amount of the observed variance in the response variable is unexplained by the model. This variance introduces an uncertainty in prediction even when the range of variation in the regressor variable is within the range over which the model was identified. The next section of this paper addresses this source of uncertainty which is probably the most important.

(b) Model mis-specification errors which are due to:

i) inclusion or non-inclusion of certain regressor variables. Usually secondary effects such as humidity or solar radiation are either neglected or assumed to manifest themselves along with other variables which appear explicitly in the model;

ii) assumption of a linear model, when the physical equations suggest non-linear interaction among the regressor variables;

iii) incorrect order of the model, i.e., either a lower order or a higher order than the physical equations suggest.

Physical interactions of the system will dictate the model structure and statistics by itself may be of limited use. As stated earlier, the physics of energy use in buildings is well-known, which, when coupled with a large body of previous experience, suggest that this source of uncertainty is probably not very influential in statistical modeling of building energy use.

(c) **Model extrapolation errors** which arise when a model is used for prediction outside the region covered by the original data from which the model has been identified. An illustration of this error is when a whole-building chilled water use model is developed using data exclusively from the winter months when energy use is low and when the range of variation in ambient temperature does not adequately cover an entire possible yearly range of variations. Error due to model extrapolation is a serious concern in estimating building energy retrofit savings in the LoanSTAR program because in many buildings the pre-retrofit period does not span an entire year. This issue is currently under investigation.

(d) **Improper residual behavior**. Major assumptions during regression are that the residuals have:

i) zero mean;

ii) constant variances, i.e., heteroscedasticity is not present;

iii) are uncorrelated, i.e., no serial correlation or autocorrelation is present; and,

iv) a near-normal distribution.

The method of least squares can be used to estimate the parameters in a linear regression model regardless of the form of the distribution of errors, and so the last assumption is not relevant in our current savings calculation methodology. Assumption (i) is also not a serious criterion because it is satisfied in most cases. The normal manner to deal with heteroscedasticity is to perform a weighted regression with the observations inversely weighted with their variance (Draper and Smith 1981). Data from the LoanSTAR buildings do not seem to generally exhibit heteroscedasticity, and consequently this issue will be overlooked in our current discussion.

Autocorrelated residuals may arise due to two primary reasons, (1) model mis-specification, and (2) autocorrelation in the regressor variable itself. The first, which clearly indicates an inadequate model identification process needs to be resolved from physical considerations. The second cause of autocorrelated residuals is because standard regression assumes the predictor variable to be a set of random data. However, continuous or time series data averaged over daily time scales may still retain information from previous days (i.e., data are not entirely random), and the serial correlation in the regressor variables is subsequently transmitted to the residuals.

There are several ways of analyzing time series data in a pure regression framework (see Neter et al. 1989). One way is to explicitly incorporate the effects of the autocorrelation into the error analysis itself (see Thiel 1971, Pindyck and Rubenfeld 1981). Another way, and this is the one adopted later in our analysis, is to transform the set of autocorrelated data into another set of random data wherein autocorrelation effects have been removed. The practical implication of neglecting serial correlations in the data is that equations presented in elementary statistical textbooks for model prediction uncertainty of random data will differ from the true model uncertainty. Generally, the issue of serial correlations seems to have been overlooked by building energy data analysts, and it is one of the primary objectives of this paper to explicitly point this out and discuss means of addressing this issue.

A final point of discussion is multicollinearity, i.e., the regressor variables are correlated to one another. The lack of perfect independence among the regressor variables has serious consequences (Pindyck and Rubenfeld 1981; Manly 1986):

(i) in terms of clouding the interpretation that regression coefficients yield on how the response variable is affected with unit change in the particular regressor variable, and

(ii) rendering the estimated values of the coefficients very sensitive to slight changes in the data, and thereby predicting unphysically large standard errors of the estimate.

Techniques to overcome these limitations are available and one of these, namely the Principle Component Analysis, has already been applied to model energy consumption in a supermarket (Ruch et al. 1991). In the case of commercial buildings in the LoanSTAR program, multicollinearity does not seriously affect the estimates of model prediction errors. The change-point models are currently limited to regression models with one regressor variable and so the question of multicollinearity does not arise. For linear multiple regression models, electricity use due to lights and appliances is often not a significant regressor variable, and even when it is, it is generally poorly correlated with ambient temperature on a daily time scale. Thus, multicollinearity effects among the regressor variables will have little, if any, bearing on the estimates of retrofit savings uncertainty in the LoanSTAR program. Nevertheless, the issue of multicollinearity may need to be satisfactorily addressed in the future when model identification reaches a higher level of sophistication.

The various sources of errors and uncertainties discussed above lead to either **random** errors or systematic errors. A **systematic** error results in a uniform bias in some variable while random errors have no regular pattern. Systematic errors could arise due to improper calibration and installation of metering equipment. The former type of systematic error can be compensated for at a later stage while the latter cannot. In terms of the statistical estimation, model-mis-specification and model extrapolation could result in biased predictions while improper attention to residual behavior could result in estimates of prediction uncertainty being lower than those actually present.

Removing or minimizing systematic bias in model identification cannot be done based on statistical grounds alone. Taking care to include all the physical interactions of the system and to verify that the chosen model structure is physically consistent, will minimize model mis-specification errors during the model identification process. Allowing for a sufficiently long pre-retrofit period on which to base the model identification process will minimize model extrapolation errors. Treatment of errors in statistical textbooks normally presumes no systematic bias, both in the measurement stage and in the model identification process. Thus, only random error behavior can be adequately treated in a statistical framework.

Model Prediction Uncertainty

In this section, we shall assume only random measurement errors to be present in our data and present equations for deducing the uncertainty bounds of our estimates of retrofit energy savings due to **model prediction errors**, without and with autocorrelations present in the data.

With Random Data

Consider the case when observed pre-retrofit data of energy consumption in a commercial building support a linear regression model with no change point behavior, as follows:

$$\hat{E}_i = a_o + a_1 * T_i \tag{2}$$

where

- T = daily average ambient dry bulb temperature,
- $\hat{E}$ = daily total energy use predicted by the model
- i = subscript representing a particular day over the pre-retrofit period, and,
- a_o and a_1 are the least-square regression coefficients.

Once a regression equation has been identified, it can be used for forecasting purposes, i.e., to predict $\hat{E}_{Pred,j}$ values under specified future conditions of T_j. This prediction will, however, have a prediction uncertainty associated with it, which in statistical terms, is quantified by a prediction variance (Draper and Smith 1981). At this juncture, let us mention that most statistical textbooks presume that predictor variables in a regression model have no measurement error. Though this is seldom true, it avoids the need for a much more complicated statistical treatment. We shall assume this to be true in our analysis procedure as well.

The prediction uncertainty of a simple linear model identified from random data, i.e., without autocorrelations being present, is straightforward and is given in most statistical textbooks (Draper and Smith 1981; Pindyck and Rubenfeld 1981). The prediction uncertainty for an INDIVIDUAL observation during the post-retrofit period, neglecting autocorrelation effects, is:

$$\sigma^2(\hat{E}_{Pred,j}) = S^2(\hat{E}_i)\left[1 + \frac{1}{n} + \frac{(T_j - \bar{T}_n)^2}{SS_T}\right] \tag{3}$$

where $S^2(\hat{E}_i)$ is the mean square error during the pre-retrofit period computed as:

$$S^2(\hat{E}_i) = \left[\frac{1}{n-(k+1)}\right]\sum_{i=1}^{n}(E_i - \hat{E}_i)^2 \tag{4}$$

where

- n = number of days of pre-retrofit period
- k = number of regressor variables in the model
- $\bar{T}_n$ = mean value of T_i during the pre-retrofit days
- SS_T = sum of squares of T_i, computed as

$$SS_T = \sum_{i=1}^{n}(T_i - \bar{T}_n)2 \tag{5}$$

The second term within brackets in equation (3) accounts for the variance in predicting the **mean** $\hat{E}_{Pred,j}$. value for a given T_j value. However, because each post-retrofit day has a different T_j value, the prediction variance has to be increased by $S^2(\hat{E}_i)$, which accounts for the unity within the brackets. The last term in the brackets accounts for the increased uncertainty when the prediction is made at a point other than the centroid of the pre-retrofit data used to develop the model. The retrofit savings methodology is not, however, based on individual predictions of $\hat{E}_{Pred,j}$. We are more interested in the sum over m days of $\hat{E}_{Pred,j}$ values rather than on any one individual day.

Subsequently, assuming

$$\sum_{j=1}^{m}\sigma^2(\hat{E}_{Pred,j}) = \sigma^2\left(\sum_{j=1}^{m}\hat{E}_{Pred,j}\right) \tag{6}$$

the total prediction variance can be obtained from equation (3) as:

$$\sigma^2\left(\sum_{j=1}^{m}\hat{E}_{Pred,j}\right)$$

$$= \sigma^2(\hat{E}_{Pred,Tot}) \tag{7}$$

$$= S^2(\hat{E}_i) * m^2 * \left\{\frac{1}{m} + \frac{1}{n*m} + \frac{\sum_{j=1}^{m}(T_j - \bar{T}_n)^2}{m^2 * SS_T}\right\}$$

A look at equation (7) reveals that there are basically three factors which contribute to the total prediction uncertainty:

(a) finite number of post-retrofit days used to predict pre-retrofit energy consumption,

(b) finite numbers of pre-retrofit days used to identify the regression model,

(c) uncertainty due to prediction away from the centroid of pre-retrofit data set (i.e., away from $\overline{T}_n$).

We note that increasing n and m results in an effective relative decrease in the prediction uncertainty which is intuitively obvious.

The energy savings and associated uncertainty (ϵ) on any particular day j during the post-retrofit period are:

$$\left(E_{Save,j} \pm \epsilon_{Save,j}\right) = \left(\hat{E}_{Pred,j} \pm \epsilon_{Pred,j}\right) - \left(E_{Meas,j} \pm \epsilon_{Meas,j}\right) \tag{8}$$

Using the standard method of combining random errors (Schenck 1968), we have

$$\epsilon_{Save,j} = \left[\left(\epsilon_{Pred,j}\right)^2 + \left(\epsilon_{Meas,j}\right)^2\right]^{1/2} \tag{9}$$

The total energy savings over m days from equation (1) is:

$$\left(E_{Save,Tot} \pm \epsilon_{Save,tot}\right) = \left(\hat{E}_{Pred,Tot} - E_{Meas,Tot}\right) \pm \epsilon_{Save,Tot} \tag{10}$$

with

$$\epsilon_{Save,Tot} = \left[\sigma^2\left(\hat{E}_{Pred,Tot}\right) + m^2 * \sigma^2\left(E_{Ins}\right)\right]^{1/2} \tag{11}$$

where $\sigma^2(E_{Ins})$ is the absolute instrumentation error assumed constant and taken as independent of the relative magnitude of actual observed data and instrument full-scale reading. (This approximation is valid because the error in individual measurements is often quoted by the manufacturer as a fraction of the full scale meter reading.)

We would like to point out that measurement errors (assumed random) present in the pre-retrofit data are implicitly contained in the variance of the pre-retrofit energy use data and should not be included again.

Equation (11) gives an estimate of the absolute uncertainty in estimated energy savings. It may be more meaningful to express the savings uncertainty as a relative error (RE) defined as:

$$RE = \frac{\epsilon_{Save,Tot}}{E_{Save,Tot}}$$

In the case of no instrument error, the following expression is obtained from equation (7), equation (10) and equation (11)

$$RE = \frac{S\left(\hat{E}_i\right) * \left\{\frac{1}{m} + \frac{1}{n*m} + \frac{\sum_{j=1}^{m}\left(T_j - \overline{T}_n\right)^2}{m^2 * SS_T}\right\}^{1/2}}{<\left(\hat{E}_{Pred,j} - E_{Meas,j}\right)>} \tag{12}$$

where $<(\hat{E}_{Pred,j} - E_{Meas,j})>$ is the average daily savings during the post-retrofit period. Note that it suffices to use the model once with the mean T_j value in order to get the mean daily predicted energy use.

The normal statistical practice is to specify the confidence interval and not the uncertainty itself. Often confidence intervals of 95% of a two-tailed distribution, are selected. The total measured energy savings and the confidence intervals are given by (see any appropriate statistics book, for example, Neter et al. 1989):

$$E_{Save,Tot} \pm \left(\epsilon_{Save,Tot}\right) * t\left[n - (k+1), 1 - \frac{\alpha}{2}\right] \tag{13}$$

where

$t\left[n - (k+1), 1 - \frac{\alpha}{2}\right]$ is the t-statistic tabulated in most statistical textbooks,
α is the significance level, and
k is the number of independent variables in the regression model.

After four months of pre-retrofit data, t asymptotes to 1.96 at a confidence level of 0.95.

With Autocorrelated Data

As stated earlier, analysis of the LoanSTAR data is done on a daily time scale where strong serial correlations are present in the basic data set. This results in autocorrelated residuals in the model predictions. The remedial approach adopted here to overcome the serial correlation effects is to transform the original data set so that the residuals have no serial correlation. There are several techniques of doing so, and we shall resort to the widely used Cochrane-Orcutt procedure (Neter et al. 1989; Pindyck and Rubenfeld 1981) which, in essence, is a first-order auto-regressive (AR) scheme.

Let us first recall the definition of the autocorrelation coefficient at lag 1 of a time series data stream, say x_1, x_2, ...x_n:

$$\rho_1(x_i) = \frac{cov(x_i, x_{i+1})}{\sigma^2(x_i)} = \frac{\sum_{i=1}^{n-1}(x_i-\bar{x})(x_{i+1}-\bar{x})}{\sum_{i=1}^{n-1}(x_i-\bar{x})^2} \quad (14)$$

where
$\bar{x}$ is the mean value of x_i,
cov is the covariance operator, and
σ^2 is the variance operator.

A value of $\rho_1 = 0$ indicates no autocorrelation, while $\rho_1 = 1$ represents perfect autocorrelation. Though ρ can have negative values, we find that in our case, values of ρ are invariably positive and in the range 0.4 - 0.95.

Consider the regression model given by equation (2). The Cochrane-Orcutt procedure involves transforming the basic variables as follows:

$$E_i' = E_{i+1} - \rho_1 * E_i, \text{ and} \quad (15)$$
$$T_i' = T_{i+1} - \rho_1 * T_i$$

Strictly speaking, the autocorrelation coefficient ρ_1 should be that of the residuals of the regression model. Because this is not known prior to the regression itself, the Cochrane-Orcutt procedure proposes that the initial value of ρ_1 be estimated by classical least-squares regression and that iteration be done until the E_i' and T_i' data streams are rendered random. (A Durbin-Watson test can be done to verify randomness.) Our preliminary analysis on LoanSTAR data has indicated that such an iteration is usually not necessary and that a single transformation using the autocorrelation coefficient of the least-square regression residuals is adequate.

Next, the transformed data are regressed and the following model identified:

$$\hat{E}_i' = a_o' + a_1' * T_i' \quad (16)$$

where a_o' and a_1' are the least-square regression coefficients.

Finally, a model in the original variables is obtained by a back transformation of the regression coefficients:

$$\hat{E}_i = b_o + b_1 * T_i \quad (17)$$

with $b_o = \dfrac{a_o'}{1-\rho_1}$ and $b_1 = a_1'$

Let us now address the issue of estimating the uncertainty in our retrofit savings in the presence of serially correlated data. One approach is to simply use the equations presented in the previous section along with the transformed data set. A physically more appealing procedure is to derive an expression for the uncertainty in terms of the original data set. This would allow greater flexibility and enhance our intuitive understanding of the degree to which autocorrelations impact model prediction uncertainty. This problem does not seem to have been treated previously, and consequently we shall derive the solution from first principles.

The model given by equation (16) can be "centered" and written as:

$$\hat{E}_1' = \bar{E}_n' + a_1' * (T_i' - \bar{T}_n') \quad (18)$$

where $\bar{E}'_n$ and $\bar{T}'_n$ are the mean values of $\hat{E}'_i$ and T'_i over n observations.

From basic statistics and assuming no measurement error in T_i, the variance of the predicted mean value of E' at a specific value of T'_o is:

$$\sigma^2(\hat{E}_o') = \sigma^2(\bar{E}_n') + (T_o' - \bar{T}_n')^2 * \sigma^2(a_1') \quad (19)$$

The standard expression for $\sigma^2(a'_1)$ is given in statistical textbooks (p. 24 - 28 of Draper and Smith 1981). Setting $\rho_1 = \rho$, equation (19) can be written as:

$$\sigma^2(\hat{E}_o') = \sigma^2(\hat{E}_i')*$$

$$\left[\frac{1}{n-1} + \frac{(T_o'-\bar{T}_n')^2}{\sum_{i=1}^{n-1}(T_i'-\bar{T}_n')^2}\right] \tag{20}$$

For an AR1 process, from Box and Jenkins (1976), p. 58

$$\sigma^2(\hat{E}_o') = \sigma^2(\hat{E}_o)*(1-\rho^2) \tag{21}$$

We note that, for large values of n,

$$\sum_{i=1}^{n-1}(T_i' - \bar{T}_n')^2 = \frac{SS_T}{1-\rho^2} \tag{22a}$$

where SS_T is given by equation (5),

and

$$(T_o'-\bar{T}_n')^2 \cong (1-\rho)^2(T_o-\bar{T}_n)^2 \tag{22b}$$

Also,

$$\sigma^2(\hat{E}_i') \cong \frac{S^2(\hat{E}_i)}{1+2\rho^2} \tag{23}$$

where $S^2(\hat{E}_i)$ should be evaluated from equation (4) with $\hat{E}_i$ values computed from equation (17). (The interested reader can refer to p. 255 of Thiel 1971, for a complete derivation.) Because the regression lines of the original data (given by equation 2) and of the back-transformed data (given by equation 17) are usually close (this aspect is discussed in the case study in the next section), it is easier to estimate $S^2(\hat{E}_i)$ from equation(4) with $\hat{E}_i$ values computed from equation (2).

Introducing the above equations into equation (20):

$$\sigma^2(\hat{E}_o) \cong \frac{S^2(\hat{E}_i)}{(1-\rho^2)*(1+2\rho^2)}*$$

$$\left[\frac{1}{n-1} + (1-\rho^2)*(1-\rho)^2\frac{(T_o-\bar{T}_n)^2}{SS_T}\right] \tag{24}$$

Equation (24) is valid for the mean prediction uncertainty. For an individual observation, the prediction variance is given by:

$$\sigma^2(\hat{E}_{Pred,j}) \cong \frac{S^2(\hat{E}_i)}{(1-\rho^2)*(1+2\rho^2)}*$$

$$\left[1 + \frac{1}{n-1} + (1-\rho^2)*(1-\rho)^2\frac{(T_j-\bar{T}_n)^2}{SS_T}\right] \tag{25}$$

Equation (25) is the counterpart of equation (3) when serial correlations are present. Proceeding similarly to that outlined in the previous section, the expression for relative error in the retrofit savings in the absence of measurement errors is deduced:

$$RE = \frac{1}{<(\hat{E}_{Pred,j} - E_{Meas,j})>} * \frac{S(\hat{E}_i)}{\{(1-\rho^2)*(1+2\rho^2)\}^{1/2}}$$

$$\left\{\frac{1}{m} + \frac{1}{m*(n-1)} + (1-\rho^2)*(1-\rho)^2*\frac{\sum_{j=1}^{m}(T_j-\bar{T}_n)^2}{m^2*SS_T}\right\}^{1/2} \tag{26}$$

Note that the main difference in the RE values from equation (12) and equation (26) are caused by the presence of the term $\{(1-\rho^2)*(1+2\rho^2)\}^{1/2}$ in the denominator of equation (26). This factor can be greater or less than 1 depending on the value of ρ. Values of ρ less than about 0.75 will result in uncertainty of the serially correlated data to be slightly less than that (less than 5%) of uncorrelated data. For values of $\rho > 0.75$, the opposite holds true with the difference increasing sharply for values of ρ approaching unity.

Equation (26) is analogous to equation (12) when autocorrelations are present. The above derivation is subject to several assumptions which need explicit mention:

(a) no measurement error in T, both during the pre- and post-retrofit periods,

(b) there is no systematic bias in the measurements,

(c) the energy data is homoscedastic, i.e., has constant variance,

(d) autocorrelation in the residuals is not due to model misspecification errors,

(e) the first-order autocorrelation coefficients of energy use and temperature data streams are close,

(f) there is no autocorrelation in the post-retrofit measurements of E_j.

Of the above assumptions, (a) and (f) need to be addressed at a later date while the other assumptions are probably not serious ones where LoanSTAR data is concerned. Extending the above equations to linear segmented regression models, i.e., change-point models, is complex and is currently being studied. However, if we assume (i) that the change point is known without any inherent uncertainty, and (ii) that the autocorrelation effects on either side of the change point are similar, we can simply take the data as falling in two separate regions and treat the overall model as made up of two individual simple linear models, the uncertainty of each being determined along the lines discussed above. The individual variances are then added in quadrature to yield the total variance and hence the total uncertainty.

Case Study

In this section, we shall apply our equations for estimating uncertainty in retrofit savings to data from a LoanSTAR building. We shall overlook uncertainty in our measurements of post-retrofit data and study the effects of model prediction uncertainty in our saving estimates.

The building is a large engineering center in Central Texas with a gross floor area of 324,000 ft^2. It is open 24 hours per day and 365 days per year and though it exhibits marked weekday and weekend differences, we shall overlook this difference in this illustrative study. The pre-retrofit monitoring period included about 400 days of good data, a data length long enough for satisfactory regression model identification. The scatter plots of daily chilled water use versus ambient dry bulb temperature is shown in Figure 1(a). We note that this plot does not exhibit change-point behavior and a linear regression model is adequate.

Table 1 gives various statistics relevant for retrofit savings calculations. The classical least-square regression model (equation 2) fits the data well ($R^2 = 0.84$) as can be seen from Figure 1(a). The autocorrelations are strong ($\rho = 0.92$) for both energy use and ambient temperature. The post-retrofit period spans 380 days. We note that mean values of ambient temperature during pre- and post-retrofit periods are 69.1°F and 72.7°F, respectively, while the corresponding chilled water energy use values are 132.5 and 80.9 MBtu/day respectively, a distinct difference. From Table 1 we also note that ρ of the residuals using classical least squares is 0.587, with the Durbin-Watson statistic of 0.83 indicating strong serial correlation. The data streams were transformed using $\rho = 0.587$ and the new regression line thus obtained has an R^2 of 0.63 and a Durbin-Watson statistic of 1.89 indicating no serial correlation. The regression line (equation 17) considering autocorrelation is also shown in Figure 1(a).

Figure 1(b) illustrates the fact that the post-retrofit daily chilled water use data points fall well below the linear regression line which models pre-retrofit daily use. The vertical difference between this line and the data points represent daily savings, as is obvious from equation (1). The 95% uncertainty bands estimated from equation (13) and equation (25) are also shown. These bands are about $\pm$ 19.6 MBtu/day, i.e., $\pm$ 14.8% of the mean daily energy use during the pre-retrofit period. These bands turn out to be close to the corresponding bands assuming no serial correlation effects, i.e., using equation (3) and equation (7). This is due to the particular value of ρ in the data set as discussed earlier. The scatter plot in Figure 1(c) shows how daily savings vary with ambient temperature. As stated earlier, the building exhibits distinct differences in weekday and weekend operation (which we have chosen to overlook here) causing the data points in both Figures 1(b) and 1(c) to fall along two distinct lines.

How the measured daily savings and the associated 95% confidence intervals vary from day-to-day can be seen in the time series plots shown in Figure 1(d). Of more interest is how the cumulative or total chilled water savings and the associated confidence intervals vary with time from the day the retrofits were completed. From Figure 1(e), we see that the savings keep increasing monotonically as does the absolute or total uncertainty, testified by the slight widening of the confidence band with time.

Finally, how the 95% relative error or relative uncertainty given by the product of the t-statistic (equation 13) and equation (26) varies with day number m is shown in Figure 1(f). We note that the relative error drops sharply and asymptotes to a value close to 3%. After about three months into the retrofit period, the relative error seems to have more or less stabilized. This suggests that one should allow at least 3 months of post-retrofit data in order for the reported retrofit savings to be sound.

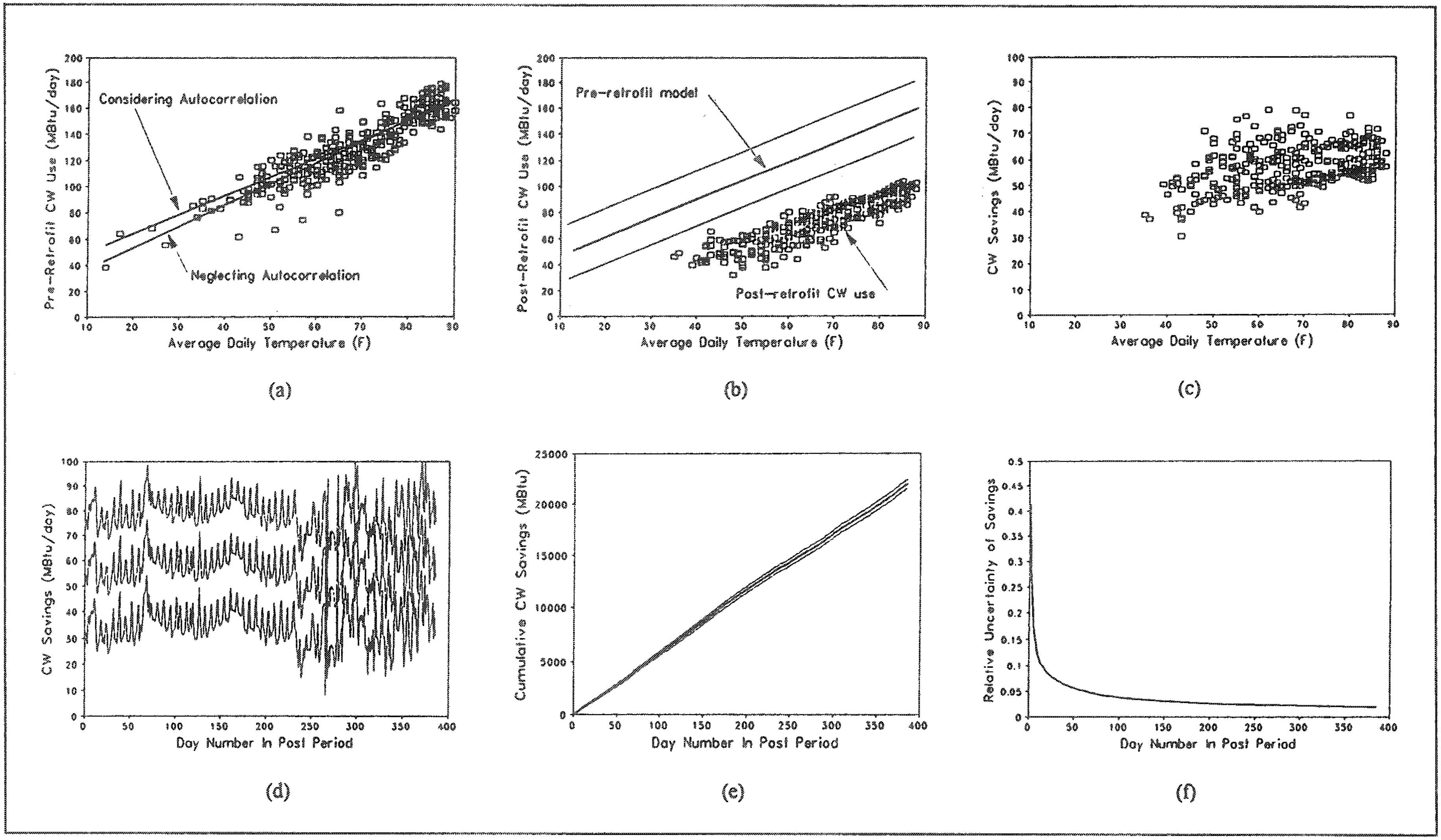

Figure 1. Series of Plots of Chilled Water Use in the Case Study Building Demonstrating:

a) pre-retrofit chilled water use data points and regression lines neglecting and considering autocorrelation effects,
b) pre-retrofit model along with the 95% confidence bands, and post-retrofit chilled water use data points. The vertical distance between the model and a data point is the chilled water savings on that day,
c) daily chilled water savings plotted against average daily outdoor air temperature,
d) daily chilled water savings and uncertainty bands plotted against time,
e) cumulative chilled water savings and uncertainty plotted against time, and
f) the relative uncertainty of retrofit savings plotted against time.

Table 1. Statistics of Daily Chilled Water Use in the LoanSTAR Building

			Chilled Water
Pre-retrofit model goodness of fit with untransformed data		$\overline{R}^2$	0.84
Number of pre-retrofit days		n	399
Mean Square Error of pre-retrofit energy use with untransformed data		$S^2(\hat{E}_i)$	110.0
Mean pre-retrofit values:	Energy Use	$\overline{E}_n$	132.5
	Ambient Temperature	$\overline{T}_n$	69.1
Sum of squares during pre-retrofit of temperature		SS_T	86270
Autocorrelation coefficient:	Energy use	ρ	0.928
	Ambient Temperature		0.915
	Residuals		0.587
Pre-retrofit model goodness of fit with transformed data		$\overline{R}^2$	0.63
Durbin-Watson statistic:	Untransformed data		0.83
	Transformed data		1.89
Number of post-retrofit days:		m	380
Mean post-retrofit values:	Energy Use	$\overline{E}_m$	80.9
	Ambient Temp.	$\overline{T}_m$	72.7
95% Prediction uncertainty of mean daily use:			
	Neglecting autocorrelation		20.6
	Considering autocorrelation		19.6

Summary

The main objectives of this paper were two-fold:

i) to discuss the sources of errors present in the regression approach of modeling building energy use in general, and in the LoanSTAR project in particular; and highlight the fact that serial correlation effects in the time series data need to be explicitly considered during the model identification stage itself, and;

ii) to present the statistical equations for predicting uncertainty or confidence intervals of estimated retrofit energy savings due to model prediction uncertainty in the presence of serial correlations, and to illustrate the use of these equations with a case study using data from a LoanSTAR building.

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Nomenclature

a_o, a_1	Least squares regression coefficients of the original data
a'_o, a'_1	Least squares regression coefficients of the transformed data
b_o, b_1	Model coefficients of the original data obtained by back transformation
CV	Coefficient of variation
E	Energy use

k	Number of regressor variables in the model
m	Number of post-retrofit days
n	Number of pre-retrofit days
RE	Relative Error
R^2	Coefficient of determination
S^2	Mean square error
SS	Sum of squares
T	Ambient dry bulb temperature
α	Significance level
ϵ	Error or uncertainty
σ^2	Variance
ρ	Autocorrelation coefficient
X'	Transformed variable of X with autocorrelation effects removed
$\overline{X}$	Mean value of X
$\hat{X}$	Model predicted value of X
$<\hat{X}>$	Mean value of model predicted value of X

Subscripts

Ins	Instrument
i	particular day during the pre-retrofit period
j	particular day during the post-retrofit period
Meas	Measured
Pred	Predicted
Save	Savings
T	Ambient dry bulb temperature
Tot	Total

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