

# Optimal Beam Daylighting with Stationary Projecting Reflector Arrays

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We have re-evaluated the prospects for implementing passive beam daylighting arrays as retrofit devices. We present a daylighting system that offers several advantages over previous designs. Our system is inexpensive, unobtrusive and can be sealed without maintenance between the lites of double-glazed fenestration. The design is based on projection theory, treating the daylighting array as a source of light that is to be projected towards a target area in a room; from this comes the name for the system--Stationary Projecting Reflector Array (SPRA). The target area towards which the light reflects from the SPRA is selected on the basis of objective criteria. For a given room's orientation, a *performance function* is derived from expressions for depth of penetration of reflected light and for line-of-sight glare from the reflections. The directional properties of the SPRA are selected by an optimization of the performance function, which simultaneously maximizes the extent of light projected into the room and which minimizes the glare throughout the year. The SPRA design thus achieves a rigorous trade-off between two principal concerns of beam daylighting. Our presentation includes derivation of the design's performance function and preliminary results from a SPRA retrofit in a classroom in Syracuse, New York.

## Introduction

A recent daylighting project (Stiles and Kinney 1991) culminated in the development of a prototype system that substantially met stringent design criteria. The criteria stated that the system should:

- Be suitable for both new and retrofit applications
- Be compatible with a variety of commercial, institutional and industrial environments
- Be both inexpensive and cost effective (payback in five years or less)
- Be easy to maintain and have a long life
- Avoid interference with curtains, blinds or drapes
- Have no moving parts, be completely passive and function without user interaction
- Provide penetration of light up to 30 feet onto the ceiling of a room
- Provide full or partial replacement of artificial light with minimal glare
- Be compatible with existing or newly-developed systems for controlling artificial light levels

- Be effective in several orientations of fenestration

Arrays of stationary reflectors comprising this daylighting system were installed in the upper third of the six windows of a classroom in Syracuse, New York, at a distance of eight feet from the floor. The purpose of this retrofit installation was to assess the potential for replacement of artificial lighting with beam daylighting. The retrofit classroom has a 30 foot by 30 foot floor area, a rectangular window area of 160 square feet and is adjacent to another classroom of the same dimensions and south-east orientation that serves as a control room for photometric studies.

On the day of installation of the retrofit in late August, light levels were measured in both the retrofit room and the control room at 11 hours, solar time. In each room, measurements were taken at nine points in a grid, such that any two adjacent points on the grid were ten feet apart, and with the outer points on the grid at a distance of five feet from the walls. The results are plotted in Figure 1, for the control room and the retrofit room.

Shown in Figure 1 are the light levels on a horizontal surface three feet from the floor (desk level). The window wall is closest to the front of the figure, where measured light levels are highest. Measurements at the back of the room are shown farthest from the window wall and they assume the smallest values. The light levels at three feet

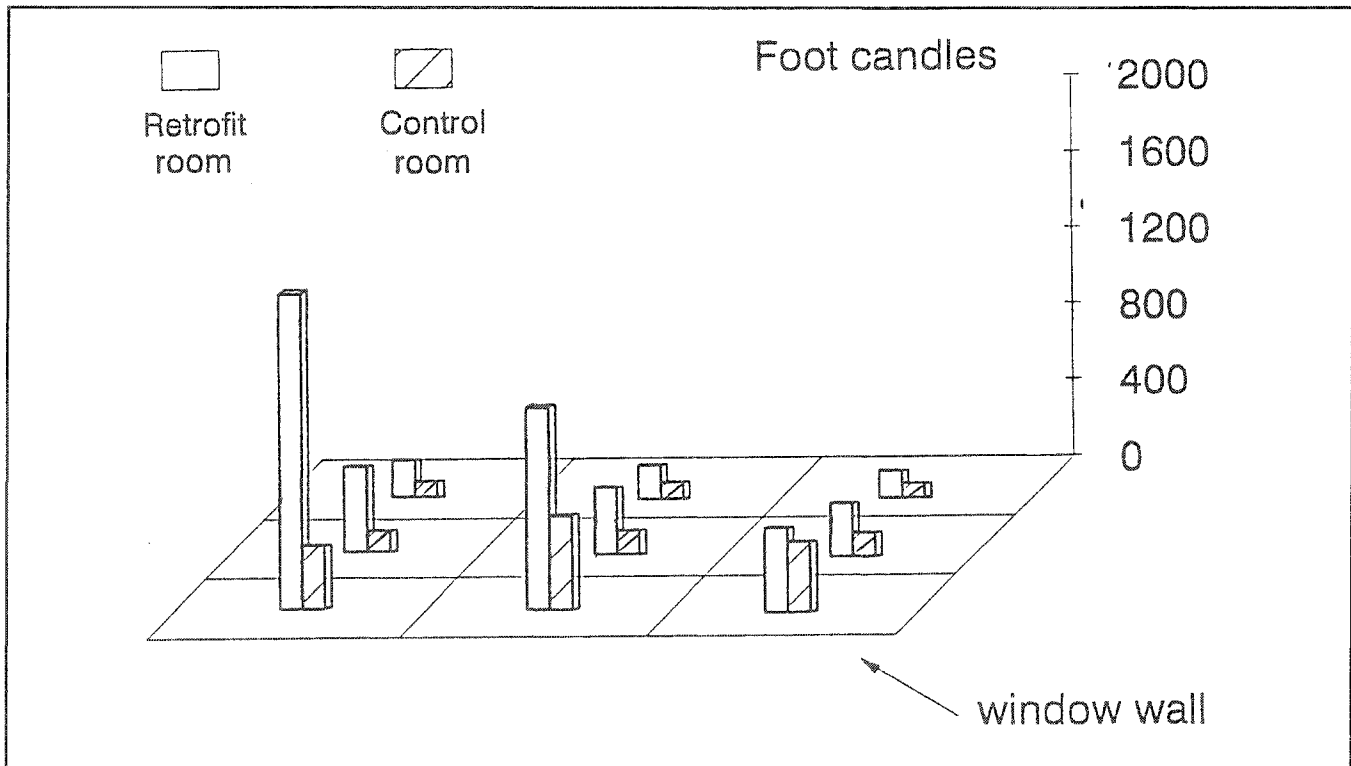


Figure 1. Light Levels in Control Room and in Retrofit Room

from the floor just inside the windows in each room were consistent at 3000 foot candles and there was no cloud cover during the interval of measurement.

The light levels seen throughout the retrofit room are much higher than light levels in the corresponding locations in the control room. Light from the daylighting arrays reflected down from the white ceiling and thereby elevated the light levels by a factor of two to four times, depending on location in the room. Even at a distance of 25 feet into the room, the average of the light levels in a row was about 140 foot candles. The preliminary results from the installed daylighting system were very encouraging on this sunny summer day.

The basic strategy that went into the design of this passive beam daylighting was to redirect sunlight towards those portions of the ceiling that spend much of the day in shadow. Note how the lowest light levels in the control room are seen for the left column of Figure 1, and that light levels in the corresponding location in the retrofit room were elevated. This intentional redirecting of reflected light was a departure from previous attempts at passive beam daylighting.

An examination of previous patents shows that while most of the designs are concerned with selective admission of

sunlight, none address the problem of what specifically is to be done with the reflected light once it enters a building (Bartenbach et al. 1987; Bar-Yonah 1985; Critten 1987; Edmonds 1991; Luboshez 1978; Morita et al. 1988; Otto et al. 1985). Further, the designs were based principally on two-dimensional ray-tracing methods, which are not systematically applicable for analysis in three dimensions.

Analysis of time-dependent reflections in three dimensions offered a methodology for meeting the design criteria listed above. Although the prototype functioned adequately in illuminating the measurably darker portions of a room, there were still problems associated with the trade-off between the convenience of stationary reflectors and the occurrence of glare at certain times of the year. The fact remains that stationary reflectors can never perform perfectly over the entire course of the year. No one, however, has sought to analyze the best possible performance of such devices.

The purpose of this paper is to present work aimed at fine-tuning the preliminary design strategy that gave the results shown in Figure 1. The paper presents:

- (1) A formal quantitative representation of the performance of stationary reflectors, in terms of the trade-off between illumination of a room and glare.

- (2) A method that finds the orientation of reflector that gives best performance at a given site latitude.

## Methods of Analysis

### The Stationary Projecting Reflector Array (SPRA)

The problem of redistributing beam daylight originates with the uneven distribution of sunlight in a room. Light levels such as those represented in Figure 1 are determined in part by the time of day and by the day of year on which the measurements are taken. The window area in the room is an architectural aperture that functions as a stop for the available solar beam. As the azimuth of the incoming beam changes with solar hour, the beam sweeps across the room in a manner not unlike that of a slowly-moving spotlight.

There are in actuality several parallels between the sweep of beam sunlight through fenestration and the operation of standard light projection devices. These parallels can be stated in the broadest of terms. In both cases, the energy of a light source must be channeled through a set of optical treatments and apertures towards an output direction. There are losses and dispersions of light flux in both types of systems. In both cases there are trade-offs between illumination and thermal problems. Although the traditional projection device has received considerably more technical attention, the spectral and thermal properties of fenestration are becoming increasingly sophisticated.

Notable differences can be found between projection systems and fenestration. The quality of image formation is not an issue in the realm of fenestration. In the case of projectors, an optical system (the condenser stage) concentrates the source of light onto the space occupied by the source of the image. No condenser stage is needed for beam sunlight, because the aperture of interest is simply the window frame. Shades, blinds and sunscreens are important adjuncts for fenestration but not for projectors. Perhaps the fundamental difference lies in the fact that traditional fenestration exerts no control over the direction of the incoming beam sunlight. The object of all projection systems is to direct as much of the available output towards a screen or at least some well defined region in space.

While there are many interesting lines of development that can come from pursuing "non-imaging projectors" as daylighting devices for fenestration, the one development treated in this paper is that of usefully redirecting beam

sunlight and projecting it into a room. Such issues as the attenuation of intensity of redirected light over distance will not be covered here.

Projection of redirected sunlight is a concept that may be applied to the old workhorse of beam daylighting, the passive array of reflectors. As mentioned in the Introduction, the previous attempts at designing passive daylighting reflectors have avoided a rigorous examination of what is to be done to beam sunlight once it is projected into a room. The general goal here is one of using a stationary array of reflectors to project sunlight towards those areas of the room that are farthest away from the windows. Accordingly, we will refer to the daylighting device as the stationary projecting reflector array, or SPRA.

The limitations of passive reflectors are many, and a rigorous examination of applying them towards a goal requires that their performance be quantified. Performance should be examined over the course of a day and for each day of the entire year. Good performance should be associated with projection of reflected light towards the back wall of a room where light levels tend to be lowest. There should be a penalty associated with line-of-sight glare due to the reflections from a given array. Taken together, these general considerations form the criteria for selecting the best possible performance that can be obtained from a SPRA.

### Qualitative Aspects of Performance

A strategy for quantifying the performance of a SPRA can be based on the sketch in Figure 2. Shown is a floor plan of a rectangular space that could be a classroom, office or similar institutional structure. At any given time of a given day of year, the incoming beam sunlight is in a specific orientation, as in Figure 2. The floor plan depicts beam sunlight going toward one end of the room while the other end of the room is relatively darker. Also shown in this figure is a desirable direction for reflected beam sunlight that would illuminate those areas in relative darkness.

Consider the case of having a reflector array located at the top section of a tall window--allowing for projection of the light into the core of the room above head height. Further, assume that the reflected light depicted in Figure 2 is parallel to the ceiling--allowing the light to penetrate to the back wall of the room. A set of reflectors oriented to give the direction of reflection depicted in Figure 2 will perform very well for a few minutes on one or more days of the year. The extent to which the reflections deviate from an ideal case can be quantified and

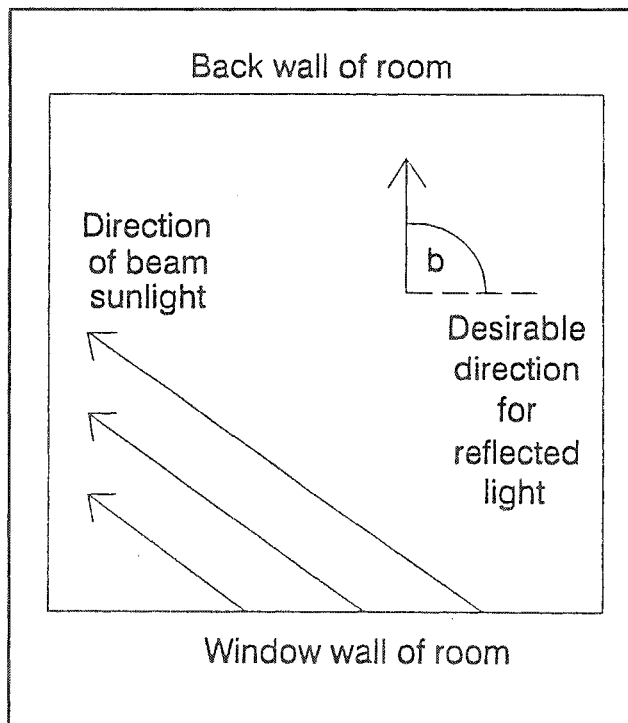


Figure 2. Penetration of Reflected Light into a Room (Floor Plan View)

simulated for purposes of evaluating the overall performance of the orientation of the reflectors.

The floor-plan angle made by the desired reflection with respect to the orientation of the inner window wall is labelled  $\underline{b}$  in Figure 2. If the angle  $\underline{b}$  is too small, the reflected light shines too close to the window wall to illuminate the core of the room. If the angle  $\underline{b}$  exceeds  $90^\circ$ , the reflected light shines in the general direction of the direct beam sunlight. The ideal case would occur if the angle  $\underline{b}$  is  $90^\circ$ , and the reflections are parallel to the ceiling; the core of the room up to the back wall would be illuminated. Hence, the angle  $\underline{b}$  can be used as one index of the penetration of reflected light into the core of the room. The other index of interest is the angle of reflection with respect to the vertical direction, which would be  $90^\circ$  when reflected light is parallel to a flat ceiling. The ideal case occurs when both angles attain  $90^\circ$ .

In order to simplify the development of performance measures, let the room depicted in Figure 2 face directly south. The problem is simplified by symmetry into two stages, one for a.m. light and the other for p.m. light. Further, the situation of beam sunlight in the a.m. case becomes the mirror-image of the situation in the p.m. case.

Treating the direct beam in Figure 2 as a case of a.m. sunlight, the performance of a reflector can be assigned a good rating on a given day if the angle  $\underline{b}$  is close to  $90^\circ$  at the time that the reflected light is parallel to the ceiling. Conversely, it can be said that performance is poor if angle  $\underline{b}$  is small when the reflected light is parallel to the ceiling. Reflector orientations that do not redirect light parallel to the ceiling at all can be considered as cases of nonperformance. Of course, performance can be considered poor if the angle of reflections with respect to the vertical direction deviates far from  $90^\circ$  as well.

A precise statement of performance for a SPRA can be made in the form of a *performance function*. A performance function appropriate for the problem has two components. One component of the performance function is a function of the angle  $\underline{b}$  as depicted in Figure 2. The other component is related to the duration of time that the reflections on a given day result in glare. A high value of performance function thus occurs for deep penetration of reflected light at such a time of day that relatively little glare ensues. *Optimal performance* can be defined as that orientation of reflector that maximizes the performance function. If performance is to be assessed for the period of an entire year, then the performance function must be computed for each day and averaged over the term of the year.

## Quantitative Aspects of Performance

*Specular Reflection of Sunlight.* The only way to quantify the directional performance of a SPRA is within a coordinate system that relates the solar beam's direction to the orientation of the reflector and to the direction of the reflected beam. The use of a coordinate system is a convenient way to embody the rules for specular reflection of the sun into the space of a room. Because it is the directional property of reflection that is of interest here, the rules for specular reflection may be imposed on a set of unit vectors. The relevant unit vectors and their placement in the coordinate system are shown in Figure 3.

The unit vectors occur in a rectangular coordinate system and their angles of position are selected in such a way as to conform to standard spherical coordinate conventions. The  $xy$ -plane is taken as parallel to the ground, with the coordinate axes pointing in the compass directions, starting with the  $+x$  axis pointing north. The  $z$  axis is perpendicular to the ground. Azimuth angle ranges from 0 to 360 degrees in the  $xy$ -plane, starting with zero degrees at the  $+x$  axis. Zenith angle is measured from the  $+z$  axis. A unit vector labelled  $s$  represents the instantaneous position of the sun with respect to the origin of the

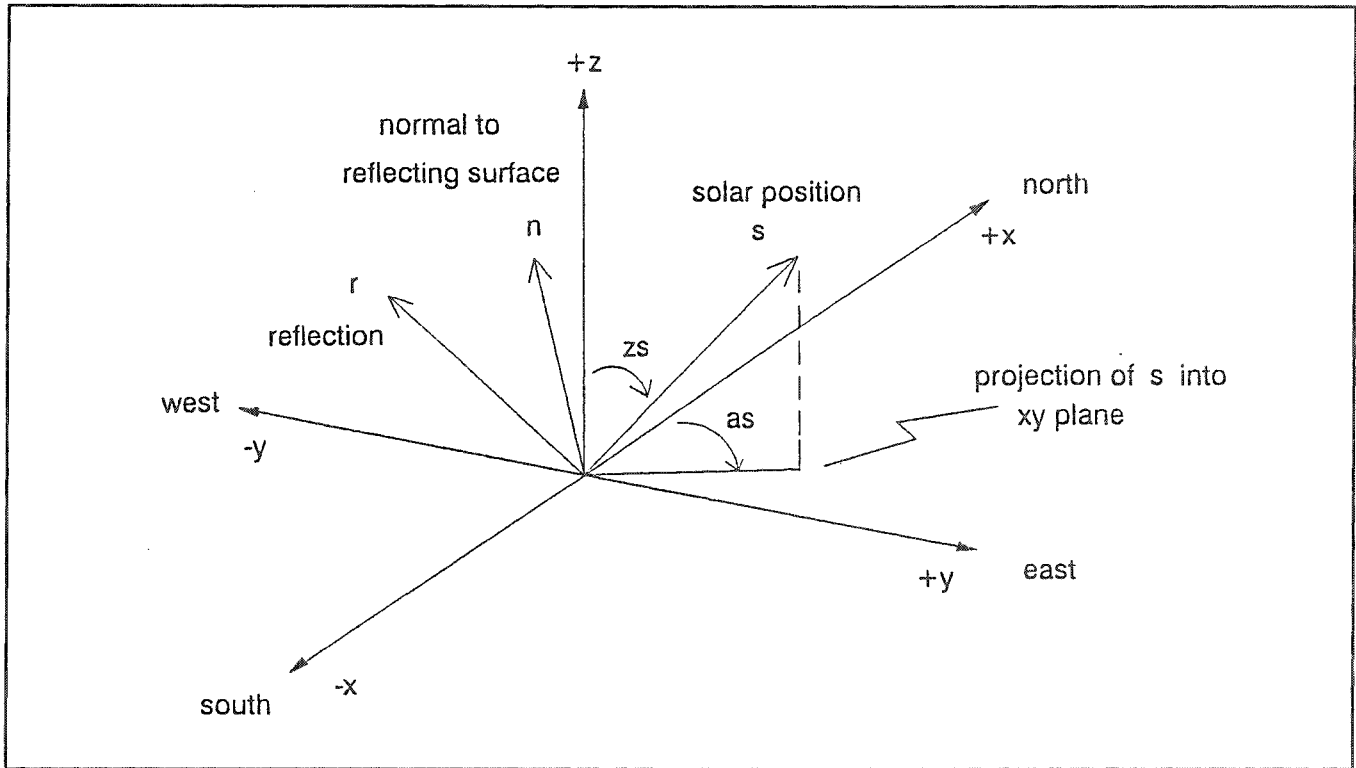


Figure 3. Vectors in a Coordinate System Useful for Beam Daylighting Design

coordinate system. The zenith angle of the vector  $s$  ( $zs$ ) and the azimuth angle of the vector  $s$  ( $as$ ) are shown in Figure 3.

The unit vector in Figure 3 labelled  $n$  represents the normal (perpendicular) vector to a patch of specular reflector. Unit vector  $r$  is the reflection of  $s$  from  $n$ . All three unit vectors have their tails at the origin of the coordinate system. The origin of the system is an arbitrary point and can be translated anywhere (without rotation) with no loss in vector directional information.

Vectors  $s$  and  $r$  are the central lines of small "pencils" of incident and reflected light, respectively, at the specular surface whose normal vector is  $n$ . It is assumed that diffuse reflection from the specular surface is low enough so that  $s$  and  $r$  are good representatives of the directions of incident and reflected light (Duffie and Beckman 1980). The direction of a unit vector can be specified by direction cosine values, of which  $s_x$ ,  $s_y$  and  $s_z$  are examples in Equation (1) for the unit vector  $s$ :

$$s = s_x i + s_y j + s_z k \quad (1)$$

The direction cosine values can be expressed in terms of a vector's zenith and azimuth angles, which for the example of the vector  $s$  are:

$$s_x = \sin(zs) \cos(as) \quad (2a)$$

$$s_y = \sin(zs) \sin(as) \quad (2b)$$

$$s_z = \cos(zs) \quad (2c)$$

Analogous expressions hold for the vectors  $n$  and  $r$ .

Vectors  $s$  and  $r$  have direction cosine values that are time-dependent. The time-varying components of the solar vector can be derived from the standard equations for the position of the sun (DiLaura 1984), and are:

$$s_x = D + E \cos(w) \quad (3a)$$

$$s_y = C \sin(w) \quad (3b)$$

$$s_z = A - B \cos(w) \quad (3c)$$

where:

$$A = \sin(l) \sin(d) \quad (4)$$

$$B = \cos(l) \cos(d) \quad (5)$$

$$C = \cos(d) \quad (6)$$

$$D = \cos(l) \sin(d) \quad (7)$$

$$E = \sin(l) \cos(d) \quad (8)$$

The variable  $\underline{l}$  in Equations (4), (5), (7) and (8) is site latitude. The variable  $\underline{d}$  in Equations (4)-(8) is the declination and is computed from Equation (9).

$$\sin d = 0.4093 \times \sin[(2\pi)(J - 81) / 368] \quad (9)$$

where  $\underline{J}$  is the Julian day of the year. Thus, the values of  $\underline{A}$  through  $\underline{E}$  will be constant for a given day of the year at a given site. The variable  $\underline{w}$  in Equations (3a)-(3c) is the hour angle and is defined as:

$$w = \frac{\pi t}{12} \quad (10)$$

Solar time  $\underline{t}$  in Equation (10) ranges from 0 hours to 24 hours.

The time-varying components of the vector  $\underline{r}$  are functions of the components of  $\underline{s}$  and  $\underline{n}$  and must be computed explicitly. A method for calculating the components of  $\underline{r}$  may be derived from the rules for specular reflection (Duffie and Beckman 1980), which in terms of Figure 3 are:

Rule (1). The angle between  $\underline{s}$  and  $\underline{n}$  is equal to the angle between  $\underline{n}$  and  $\underline{r}$ . Rule (2). The vectors  $\underline{s}$ ,  $\underline{n}$  and  $\underline{r}$  all occur in the same plane.

The quantitative formulations represented by Equations (1) through (10) are familiar to those in the daylighting field. Our original work follows, which results from applying the two general rules stated above in terms of coordinate system conventions. As an example of implementing these rules in the coordinate frame of Figure 3, assume that the components of  $\underline{s}$  and of  $\underline{n}$  are specified, and the problem is one of computing the components of  $\underline{r}$ . Denoting the angle between  $\underline{s}$  and  $\underline{n}$  as  $\underline{A}$ , the following three equations may be written using the vector dot product.

$$\begin{aligned} \underline{n} \cdot \underline{r} &= n_x r_x + n_y r_y \\ &+ n_z r_z = \cos(A) \end{aligned} \quad (11)$$

$$\begin{aligned} \underline{s} \cdot \underline{r} &= s_x r_x + s_y r_y \\ &+ s_z r_z = \cos(2A) \end{aligned} \quad (12)$$

$$\begin{aligned} \underline{V} \cdot \underline{r} &= V_x r_x + V_y r_y \\ &+ V_z r_z = 0 \end{aligned} \quad (13)$$

The vector  $\underline{V}$  in Equation (13) is perpendicular to the plane that contains  $\underline{s}$ ,  $\underline{n}$  and  $\underline{r}$  and has components ( $V_x$ ,  $V_y$ ,  $V_z$ ), which by definition can be found from the vector cross product having  $\underline{n}$  as one of its terms. Vector  $\underline{V}$  can be computed from the cross product of Equation (14).

$$\underline{V} = \underline{s} \times \underline{n} \quad (14)$$

The value of  $\cos(A)$  in Equation (11) can be computed directly from the dot product of  $\underline{s}$  and  $\underline{n}$ . The value of  $\cos(2A)$  in Equation (13) can then be found by using a trigonometric identity. From the given information, the only unknowns in equations (11)-(13) are ( $r_x$ ,  $r_y$ ,  $r_z$ ), and these components of the reflection vector may be found from simultaneous solution of the three equations. Note that this method may also be used to find the components of any one vector if the components of the other two vectors are given.

*Predicting the Occurrence of Glare.* The equations derived above are useful for handling the general problem of time-dependent specular reflection of sunlight in three dimensions. The same basic concepts may also be applied to calculate the time at which reflections begin to glare down into a room from a given orientation of reflector. Preliminary experiments with arrays having orientations useful for beam daylighting at windows showed that the reflections tend to start at the ceiling and go downward as the morning progresses (Stiles and Kinney 1991). Thus, the transition point at which the reflected beam starts to glare down into the room is the time at which the reflections are parallel to the floor.

When the reflected light is parallel to the floor, the zenith angle of reflection is  $90^\circ$ . The component of  $\underline{r}$  along the  $\underline{k}$  direction vanishes at that time (viz. Equation (2c)). Taken together with the fact that in general, because all three vectors are coplanar,

$$\underline{s} \times \underline{n} = \underline{r} \times \underline{n} \quad (15)$$

Equation (15) holds true if and only if the direction cosines of the vector on the left side are identical to the direction cosines of the vector on the right side. When the zenith of reflection achieves  $90^\circ$ , we obtain:

$$n_z s_y - n_y s_z = n_z r_y \quad (16)$$

$$n_x s_z - n_z s_x = -n_z r_x \quad (17)$$

$$\begin{aligned} n_y s_x - n_x s_y &= \\ n_y r_x - n_x r_y &= \end{aligned} \quad (18)$$

Vector  $r$  remains a unit vector when its zenith angle achieves  $90^\circ$ , so that the sum of the squares of its components equals unity; this fact may be combined with the results of squaring both sides of Equations (16) and (17) and upon adding the results we obtain after some simplification:

$$(2 n_x n_z) s_x + (2 n_y n_z) s_y + (2 n_z^2 - 1) s_z = 0 \quad (19)$$

The quantities  $s_x$ ,  $s_y$  and  $s_z$  may be substituted for those given in Equations (3a), (3b) and (3c) respectively, to give an equation of the form Equation (20). If we specify latitude, Julian day of year and the orientation of the reflector in question, the only unknowns left in Equation (20) are trigonometric functions of hour angle,  $w$ :

$$L1 \cos(w) = L2 \sin(w) + L3 \quad (20)$$

where

$$L1 = B (2 n_z^2 - 1) - 2 E n_x n_z \quad (21)$$

$$L2 = 2 C n_y n_z \quad (22)$$

$$L3 = 2 D n_x n_z - A (1 - 2 n_z^2) \quad (23)$$

Equations (21)-(23) contain the constants A, B, C, D and E (from Equations (4)-(8) for a given day and latitude) as well as the components of the vector  $n$ . Upon squaring both sides of the transcendental Equation (20) and simplifying, the following quadratic formula is obtained:

$$G1 X^2 + G2 X + G3 = 0 \quad (24)$$

where

$$G1 = L1^2 + L2^2 \quad (25)$$

$$G2 = 2 L2 L3 \quad (26)$$

$$G3 = L3^2 - L1^2 \quad (27)$$

$$X = \sin(w) \quad (28)$$

Taking the inverse sine of the appropriate root of Equation (24) gives the hour angle *at which the reflected light has a zenith angle of 90 degrees*. The solar hour corresponding to this condition may be found by using Equation (10). Note that if reflections do not attain a zenith angle of  $90^\circ$

on a given day for a given orientation of reflector, Equation (24) will have no real roots.

If conditions are such that Equation (24) does have real roots, then reflection zenith angle achieves  $90^\circ$  on the  $j$ th Julian day at a solar time to be called  $T_G(j)$ . This is a solar time that will be useful in the glare component of the performance function. Combining Equations (28) and (10) we obtain:

$$T_G(j) = \frac{12 [\text{Sin}^{-1}(X)]}{\pi} \quad (29)$$

In summary, the constraints of specular reflection can be used in the framework of a coordinate system to solve a number of problems involving time-dependent vectors in three dimensions. The system of equations that come out of this analysis form the basis for writing out the performance function for stationary reflectors.

#### *The Performance Function and Its Optimization.*

The first step in defining the performance function is to decide the time-of-day interval over which the daylighting device will be useful. This is a consideration that can be based on a given building's occupancy schedule, but for purposes of illustration, assume an eight-hour working day. Trends in availability of beam sunlight must be checked for the possibility of occlusion by the walls of a room. For example, preliminary calculations show that in Syracuse, N.Y. (latitude  $43^\circ$ ), the window wall of a south-facing room will not obstruct the solar beam on any day of the year between 8 solar hours and 16 solar hours (Stiles and Kinney 1991). This permits a working day of eight hours to be assumed for any day of the year, half of which occur in the a.m.

The a.m. half of the working day will be the four hours between the solar times of 8 hours and 12 hours on any given Julian day. Recall that the zenith angle of reflections from the range of orientations dealt with here is small in the early morning and increases during the course of the morning (Stiles and Kinney 1991). A little thought will show that glare begins at a time given by  $T_G(j)$  from Equation (29). Thus, overall *performance* is related to the fraction of the working morning that reflections *do not result in glare*.

The performance function is to be comprised of a temporal component and a spatial component. Let the temporal component of the performance function on the  $j$ th Julian day of the year be defined as  $G(j)$ . If  $T_G(j)$  is the solar time at which glare begins and is given by Equation (29), then  $G(j)$  may be defined as follows:

$$G(j) = \frac{T_G(j) - 8}{4} \quad (30)$$

$$\text{if } 8 \text{ h} \leq T_G(j) \leq 12 \text{ h}$$

$G(j) = 0$  under all other circumstances

Note that the numerator in the nonzero portion on the right side of Equation (30) is the time since eight solar hours that reflections *do not* result in glare on the  $j$ th Julian day of year; the denominator is the four-hour interval between the solar times of 8 and 12 hours.<sup>1</sup> The nonzero temporal component of the performance function is the fraction of the working morning that reflections do not result in glare.

The temporal component thus penalizes glare that starts early in the working day. It is bounded between 0 and 1. The heaviest penalty is associated with glare that occurs before or after the interval of 8 - 12 solar hours.

Let the spatial component be a function of angle  $\underline{b}$  as shown in Figure 2. The angle  $\underline{b}$  is a function of the azimuth angle of reflected light. Let the azimuth angle of reflection at the time that the zenith of reflection starts exceeding  $90^\circ$  be  $a_r(T_G(j))$ . Angle  $\underline{b}$  is then the complement of  $a_r(T_G(j))$ . The spatial component of the performance function is  $P(j)$ , such that:

$$P(j) = 1 - \frac{a_r(T_G(j))}{90^\circ} \quad (31)$$

$$\text{if } 0^\circ \leq a_r(T_G(j)) \leq 90^\circ$$

$P(j) = 0$  under all other circumstances

The maximum value of Equation (31) occurs on the  $j$ th day of year only if the reflections happen to point along  $0^\circ$  of azimuth at time  $T_G(j)$ , i.e., if the reflected light goes straight into the room towards the back wall. This is the ideal case of performance and cannot be expected from a fixed reflector for more than a few days in a year. If the reflected light achieves a zenith angle of  $90^\circ$  on the  $j$ th day of year but the azimuth of reflection exceeds the limits dictated in Equation (31), the result is penalized most heavily. The spatial component is bounded between 0 and 1.

The second step in defining the performance function requires that Equations (30) and (31) are computed for a reflector having a known orientation, i.e., that the vector  $\underline{n}$  must be specified. The entire process of optimization starts with an orientation of  $\underline{n}$  that produces, on a given Julian day at a given time, a reflection having a zenith angle of  $90^\circ$  and an azimuth angle between  $0^\circ$  and  $90^\circ$ .

The performance of this given orientation of  $\underline{n}$  is then found via Equations (30) and (31) for every day of the year between winter solstice and summer solstice (because the trajectory of the sun is symmetric with respect to that half-year interval). This procedure is then repeated but with a systematic change in  $\underline{n}$  as a function of Julian day, time of day and azimuth angle of reflection. The search proceeds iteratively in this way until the performances of all possible orientations of  $\underline{n}$  for all the Julian days in the half-year cycle are selected for all reflection azimuth angles between  $0^\circ$  and  $90^\circ$ .

The performance function itself is taken as the product of Equations (30) and (31) for a specific  $\underline{n}$  over the range of Julian days from one solstice to the next. The orientation of  $\underline{n}$  that maximizes the performance function for a given site latitude is then taken as the optimal orientation of a passive reflector. The optimization procedure may be stated succinctly in the following form:

Select  $\underline{n} = n_x \underline{i} + n_y \underline{j} + n_z \underline{k}$  such that

$$\underline{C} = \frac{\sum G(j) P(j)}{N_j} \quad (32)$$

is maximized.

In Equation (32)  $\underline{C}$  is the performance function for a reflector whose surface normal vector is  $\underline{n}$ . The number of days in the interval from one solstice to the next is  $N_j$ . The net performance of the reflector is thus quantified for each day of the year and is averaged over the entire year.

## Results

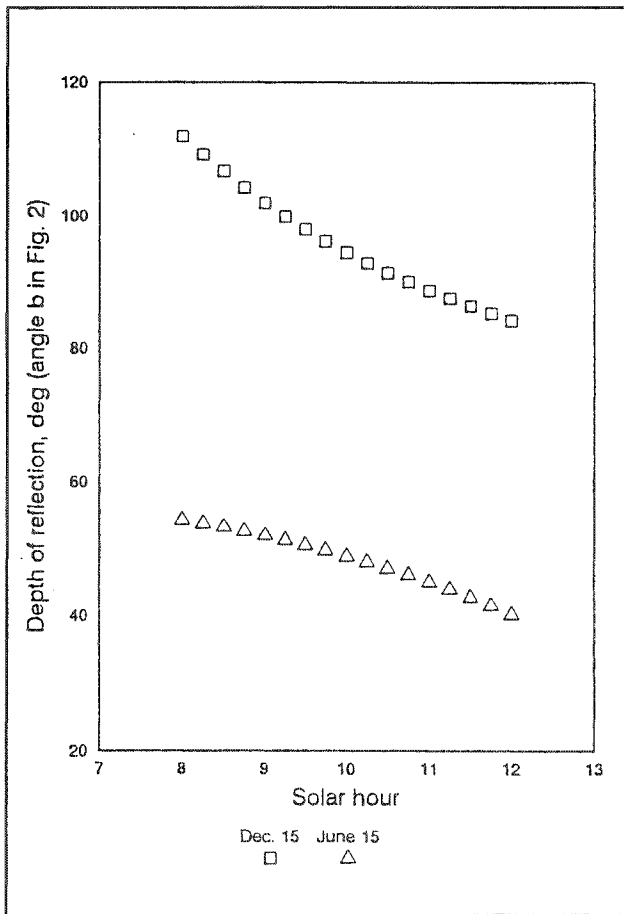
For a latitude of  $43^\circ$  (about the middle of New York State), the maximum performance function as defined in Equation (32) had a value of 0.57. The components of the normal vector  $\underline{n}$  whose simulated performance gave the maximum value of performance function are listed in Table 1. The unit vector perpendicular to the optimal orientation of reflector for this latitude has an azimuth angle of  $68.1^\circ$  and a zenith angle of  $41.2^\circ$ .

The orientation of reflector with a normal direction whose direction cosines are in Table 1 gives, on average through the year, the best trade-off between illumination into the room and glare problems. A detailed examination of the trends in reflection from the optimal  $\underline{n}$  is made in Figures 4 and 5. Those two figures are the results of modeling the direction of reflected light on the 15th of December (close to winter solstice, Julian day  $j = 349$ ) and on the 15 of June (close to the summer solstice,



**Table 1. Components of the Vector  $n$  that Maximized the Performance Function**

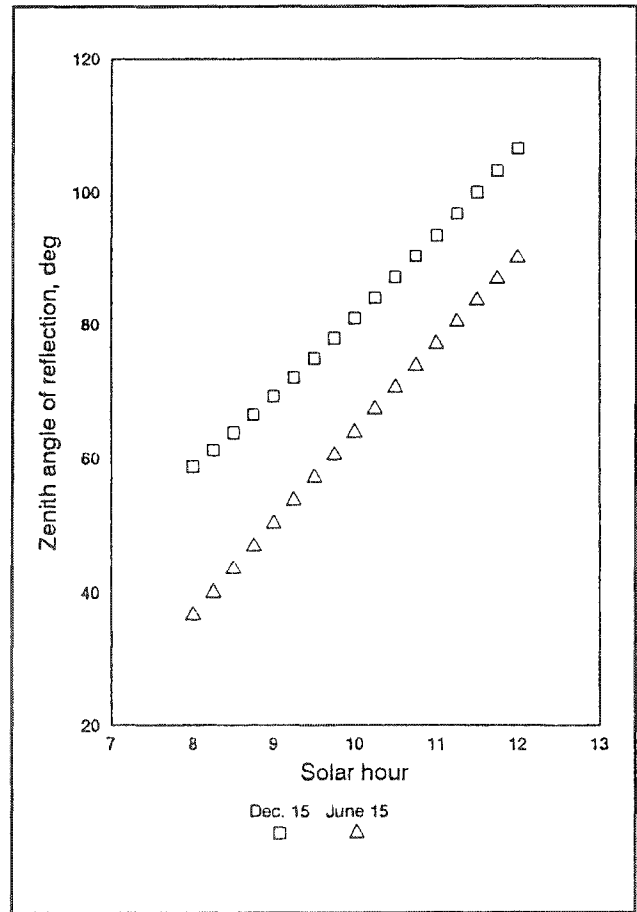
$n_x$	$n_y$	$n_z$
0.24581	0.61146	0.75213



**Figure 4. Depth of Reflected Light from Optimal Orientation of Reflector**

$j = 166$ ). Time intervals of 15 minutes are used to illustrate the trends between the solar hours of 8 to 12. The trends at intermediate dates would fall somewhere between the two curves plotted in each figure.

Depth of penetration of reflected light into a south-facing room (angle  $b$ ) is plotted against solar time in Figure 4. The value of the angle  $b$  as defined in Figure 2 was taken



**Figure 5. Zenith Angles of Reflected Light from Optimal Orientation of Reflector**

as the difference between the azimuth direction of  $90^\circ$  and the actual azimuth angle of reflection at each time. The curve for performance on Dec. 15 shows that the reflected light projects to the back wall of the room and stays there to within about ten degrees throughout the morning. On June 15, the reflections project nearly  $45^\circ$  into the room, give or take about five degrees. This is important because the sun rises early in the summer and shines in toward the west wall of the room. Reflected light that projects to the eastern wall at an angle  $b$  of  $45^\circ$  serves to balance the light levels in that season.

Zenith angle of reflection from the optimal stationary reflector is plotted in Figure 5. Note the typical pattern of reflection zenith angle being less at eight solar hours than at solar noon, with an increasing trend between those times. The only time that glare is seen to be a problem is for about an hour towards solar noon near the winter solstice. In upstate New York, this glare problem is offset

by the fact that seasonal cloud cover is most prevalent at that time of year. There is no glare problem at all by summer solstice.

## Discussion

The changing direction of usefully reflected light over the course of the morning is exemplified by the results shown in Figures 4 and 5. The overall effect is of a time-locked projection of light towards those areas of a room that are out of the range of the direct solar beam. This performance illustrates the design strategy of the SPRA daylighting device. The strategy can be implemented with flat sections of specular reflector that are oriented properly in a support frame.

Optimal performance was defined rigorously as a trade-off between depth of projection of reflected light into a room and glare. There is a minor amount of glare to contend with even in the optimized SPRA. However, the geometrical derivations given in the Methods section can be used to find *glare-free* orientations of reflectors if even a small amount of glare cannot be tolerated in a given application. The compromise would be that the redirected light would not penetrate as deeply into the room.

The actual distance that the redirected light projects into a room is a function of the azimuth and zenith angles of reflection and of the placement of the arrays. Good use can be made of indirect lighting somewhat deeper into the room by way of secondary reflections, which can be enhanced by treating the ceiling surface with a diffuse reflecting paint.

The light reflected from a SPRA does not have to penetrate to the core of a room in order to provide useful daylighting. Most of the time, the reflected light extends the bright perimeter area at the ceiling away from the windows. We are developing methods to combine simulations of beam daylighting from a SPRA system with present models of available daylight (DiLaura 1984). This is especially important at the perimeter area of a room, where diffuse daylight is most available and must be balanced with light introduced by reflectors.

A detailed prediction of light levels made available from a SPRA would be valuable in integrated lighting strategies (Matsuura 1979), whereby artificial lighting in the perimeter area can be turned off when conditions allow. In those circumstances where relatively high light levels are required, all but the deepest zones of a room can utilize the beam daylighting from a SPRA for much of a sunny day. In other circumstances, a judicious selection of

non-glaring ceiling reflectivity may even allow lighting based on SPRA alone for a portion of a sunny working day.

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## Endnote

1. The program that performs this analysis occasionally selects an orientation of reflector that can give a zenith angle of reflection at noon less than the zenith angle at eight solar hours. One example is the case of  $n$  that has an azimuth angle of zero. A subroutine checks in each case whether zenith angle of reflection at noon is greater than at eight solar hours. If not, the subroutine simply subtracts the nonzero term on the right side of Equation (30) from 1.

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