

Optimal Sizing and Control of Ice Storage and Refrigeration Systems in Commercial Buildings

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The optimal control and sizing for ice storage and chiller systems of a commercial building with climatic weather data of two US locations are presented. As an optimality criterion, the minimization of the annualized cost of borrowed capital and operating costs is chosen. This requires the solution of the optimal control problem for a range of chiller and storage capacities, generating a curve representing the edge of feasibility. The optimal solution for the chiller and storage size lies on this curve. The region around the optimal solution is explored to find zones of equal sub-optimality. Two sensitivity scenarios are presented to reveal information on the influence of design parameters.

Introduction

Cool storage systems have been promoted by electric power utilities to mitigate the supply problem of electric energy by shifting peak demands to off-peak periods, thus utilizing existing power generation resources more efficiently. For the customer, cool storage equipment presents a cost-effective measure to produce operating cost savings by using electric energy during off-peak periods at lower energy prices and demand charges.

Paramount for the effective operation of the cool storage equipment is a control scheme that minimizes the operating cost given storage size and cooling demands. Depending on the storage size, charging and discharging cycles vary and significantly affect the operating costs. While oversizing of the cool storage generally does not interfere with the design intent, undersizing does have severe implications with respect to meeting the cooling demand. As a consequence, any effective utilization of cool storage technology necessitates the solution of a coupled design and control problem for determining the size and operation protocols for the charging and discharging modes.

Manufacturers' guidelines for the sizing and equipment selection for cool storage systems ordinarily focus on simplified procedures for shifting the electric energy required for air-conditioning applications from peak to off-peak periods to take advantage of lower electricity rates during the night hours. This approach, though rather simple, has led to storage and chiller sizing procedures that resulted in workable design solutions with definite cost-saving benefits. However, this method does not

provide conclusive information about the full cost-savings potentials as a best case scenario. While optimized systems generally lack the desired reserve and, therefore are not commonly implemented, their utility lies in the possibility of comparing a practical design to what is optimally feasible. An additional motivation for investigating optimal design is derived from searching for new markets for ice storage equipment as the economics of air-condition shifts with changing electricity charges. For this particular reason it is important to know at what point ice storage becomes cost-effective and how sensitive the effectiveness of the design is to changes in the design parameters.

Several studies and design guides have addressed optimal design and control problems of cool storage facilities in a more general way by simplifying load characteristics (Musgrove 1989, 1990, Rosenfeld 1985, EPRI 1987). In Musgrove and Rosenfeld, trapezoidal load curves were applied with a sharp rise of the cooling load to values of about half to three quarters of the maximal load and a flat maximum during early afternoon hours followed by a rapid decrease to zero load in the evening. There are no building specific cooling schedules considered as a result of partial occupancy before and after regular 9am to 5pm work hours. Very early morning or late evening cooling, however, may significantly reduce the off-peak time available for recharging the storage and, thus, could impact the feasibility and economics of storage systems. From a design point of view, the EPRI design guide provides the most detail about the technology selection and

sizing process. It offers detailed assistance in determining chiller and storage capacities by applying classical load shifting approaches such as demand limiting and load leveling both of which are described for a day by day load shift. The design guide, however, fails to address the optimal control problem from a rigorous cost minimization point of view. It is paramount for the optimal design process to investigate the maximal potentials that could be realized under real operation conditions. To this end, it is necessary to focus on the control problem of a chiller/storage system as part of the design process.

Scope

In this paper, the coupled problem of the optimal design and the optimal control of cool storage will be discussed for a downtown Seattle commercial building. The paper emphasizes the optimal control problem as a fundamental method for determining the operating costs. For each selected chiller/storage combination, the optimal control problem is solved. The solution of the problem represents the minimal operating cost for a specified period.

Emphasis in this investigation is also placed on a more realistic cooling load characteristic as would be experienced in an existing building. The cooling load curves applied in this investigation were generated from a finely tuned DOE2.1d model. Although the hourly loads were not directly measured in the actual building, the simulated curves do exhibit a realistic degree of fluctuation throughout the day as generally found in buildings. An additional benefit in using a simulation program for generating load curves lies in the versatility of creating different scenarios for investigating the impacts of climatic variations or energy conservation measures on the chiller/storage selection.

Hourly cooling loads were created for two US locations, Seattle, WA and San Diego, CA. A sensitivity analysis was performed to test some design parameters for their significance in contributing to the optimal solution. In particular, the question of how much does the optimal selection change as the energy rates vary is pursued in this paper. Furthermore, three temperature scenarios were established to investigate the effect of short term temperature changes on the optimal capacity selection.

Problem Statement

To optimally size a storage/chiller system, there are two interrelated problems involving a) the optimal control of a specified chiller and storage capacity given a time dependent load profile and b) the selection of the optimal chiller

and storage capacities. Each refrigeration system with a given ratio of chiller to storage capacity has an associated optimal control solution that minimizes the operating costs over a specified period given a cooling load curve and time dependent energy charges, subject to system operating constraints such as maximal and minimal capacities. In this study, the optimal control was defined to be that control protocol which would minimize the cost over one week; this optimal relationship is referred to as "edge of feasibility." The optimal design is defined in this context as that choice of system size which when operated optimally, leads to the minimization of the sum of the annualized capital and operating costs over the period of one year.

Description of the Test Building

The building investigated in this analysis is an existing office building in downtown Seattle, Washington. An energy simulation model for this particular building using the DOE 2.1d building simulation environment was used to provide the necessary hourly cooling loads as input for the optimization. In addition to the regular simulations for a characteristic summer, intermediate, and winter period, the simulation model allowed us to investigate the effects of climatic changes on the cooling requirements. This feature was used primarily to perform three sensitivity analyses in which the thermal cooling loads due to outdoor temperature changes were investigated.

The building is a medium-rise office building with an air-conditioned floor space of approximately 350,000 square feet and a total enclosed volume of 4.8 million cubic feet extending over 9 floors including one basement level.

Most of the building's floor area is designated office space with stores and a shopping area in the first two floors. The offices are on a 8am to 5pm weekday schedule. On Saturdays the building is partly occupied for the morning hours and for portions of the afternoon. During weekdays, occupants leave the building after 5pm so that the building is virtually unoccupied after 6pm. The lighting is on a time-set schedule during regular working hours and extends until 11pm to provide lighting for the cleaning personnel.

Air supply is provided by a dual duct fan system with variable volume control systems for the upper floors while the lower floors are equipped with a constant volume system. The air distribution system is controlled according to time settings for the entire week. The primary refrigeration system consists of 3 hermetic centrifugal compression chillers of different sizes.

Description of Ice Storage and Chiller Equipment

Ice storage tanks are commonly modular, prefabricated, and fully insulated units including all or most of the internal piping. There is a variety of different unit sizes commercially available, generally ranging from 100 ton-hours (353 kWh) to 600 ton-hours (2110 kWh). A very common ice storage technology is the ice-on-coil design where plastic tubing submerged in water serves as a heat exchanger. A water/glycol solution circulates inside the tubing and extracts heat from the water to create ice on the outside of the tubing. The storage is fully charged when all liquid water is frozen solid. There are different tubing layouts possible. A common tubing configuration is a spiral form in which alternately return and supply tubing is equidistantly wrapped in a spiral shape. During the charging mode the water/glycol solution is cooled down to 26°F (-3.3°C) and produces ice at 31°F (-0.6°C). In this mode, the chiller provides a temperature which is considerably lower than produced during direct cooling where the typical chilled water temperature is about 45°F (7°C). The lower evaporator temperature has a degrading effect on the nominal chiller capacity of about 20%. The compressor efficiency, however, is only slightly reduced due to a compensating effect of the lower ambient outside air temperature at night which results in a cooler condenser water temperature. The degradation of the COP in the ice charging mode was conservatively assumed to be 20% (Musgrove 1989).

A typical air-conditioning layout is given in Figure 1. It shows the ice-storage in a parallel configuration between the chiller and cooling coils. The chiller is assumed to be equipped with a reciprocating compressor capable of

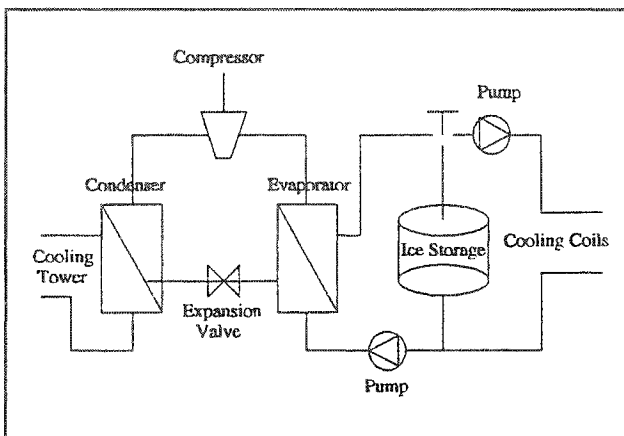


Figure 1. Typical Refrigeration Layout with Parallel Storage

running in a dual set point mode for ice storage charging as well as direct cooling. The low temperature set point during recharging requires the greatest temperature lift which reduces the coefficient of performance compared to the nominal chiller capacity.

In addition to the low temperature set point degradation, the chiller performance decreases with reducing partial load factor. A typical performance characteristic under part load conditions is shown for a reciprocating compressor chiller in Figure 2. It is assumed that the chiller will be shut off below 20% of its rated power to prevent surging.

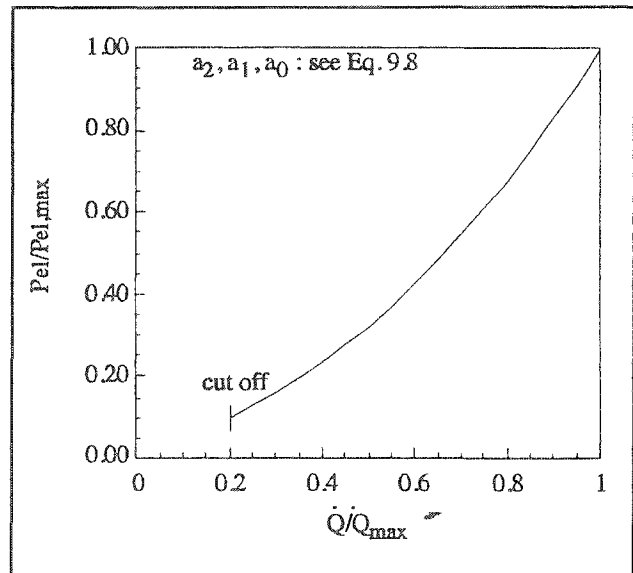


Figure 2. Non-Dimensionalized Part Load Performance Characteristic for Reciprocating Chiller. (\dot{Q}_{max} and $P_{el,max}$ are Maximal Chiller Rating and Corresponding Electrical Power.)

The capital costs of storage and chiller equipment were obtained from manufacturers approximate estimates and expressed as incremental capital cost. The economic data used for this investigation are listed in Table 1. Incremental capital costs include capital cost and labor for installation.

The maintenance costs are related to the capital costs. They are assumed to be 1% of the capital cost per year (Musgrove 1989). The life time of the chiller was conservatively estimated to be 15 years, while the storage system life time was assumed to be 25 years. The

Table 1. Economic Data for Equipment

	Storage	Chiller
Incremental Capital Cost	17 \$/kWh (60 \$/ton-hour)	171 \$/kW (160 \$/ton)
Life Time	25 years	15 years

optimization is based on the annual cost of repaying the incurred debt of the capital investment, the annual maintenance, and the operating costs (including electricity and electric demand charges). The interest rate was assumed to be 10%.

Scenarios of Optimization

Cooling Loads

The prime motivation for the analysis of a specific building was to create realistic load curve characteristics that would pose a challenge to the optimization of the design and control of ice storage and chiller systems. The load curves were generated by DOE2.1d simulation runs using Standard Meteorological Year (TMY) weather data for the two locations, Seattle and San Diego. The simulation were performed for a full year generating 8760 hourly cooling load data. The load data were then grouped into the following three seasonal periods:

- summer,
- intermediate,
- winter.

The duration of each period are different for the two locations based on the summer/winter rate schedules as set by the respective local public utility. One week in each period was then chosen as representative duration for the optimization. For the summer period, this week included the day with the maximum peak load as well as the maximum daily cooling requirement. For the intermediate and winter periods, a typical week was chosen such that

the cooling energy for that week multiplied by the number of weeks in this period equals the total cooling energy of that season.

Load profiles for the three periods are depicted in Figures 3a and 3b. Since the load curve is strongly dependent on the outside air temperature the effect of short term outdoor temperature on the chiller/storage sizing was investigated. The following three heat wave temperature profiles were assumed to extend over a one week summer period:

- upward ramping temperature,
- constant mean temperature,
- downward ramping temperature.

The daily fluctuation was approximated by a sine wave of a one day period with the highest temperature of 100°F (37.8°C) occurring at 3 pm in the afternoon. In the upward ramping temperature case the maximum temperature was reached during the last day of the week (Saturday) while in the downward ramping temperature case the highest temperature occurred during the first day. Figure 4 shows the temperature curve for the upward ramping case.

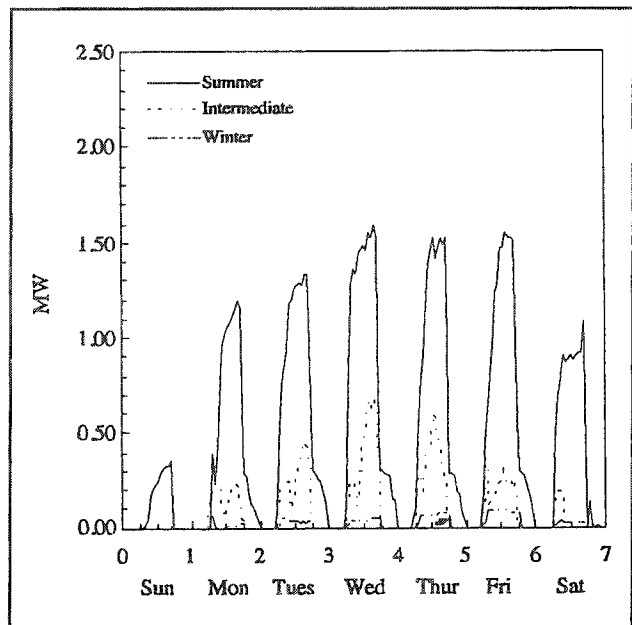


Figure 3a. Summer, Intermediate, and Winter Cooling Load Curve for Seattle

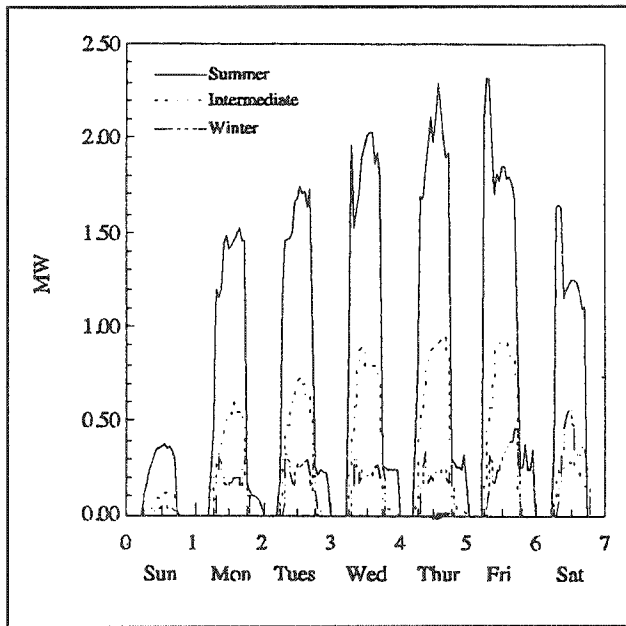


Figure 3b. Summer, Intermediate, and Winter Cooling Load Curve for San Diego

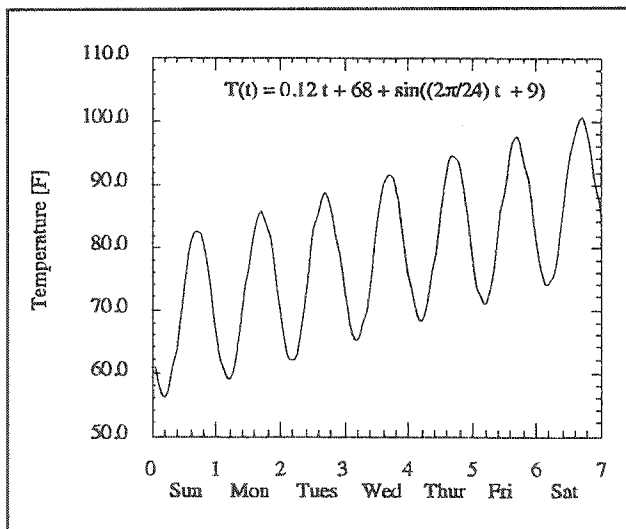


Figure 4. Upward Ramping Temperature Scenario

Electricity Rates

Both locales have the largest demand and energy charges during the period of greatest building system load. The greatest demand occurs during the heating period in Seattle, contrary to San Diego where the demand peaks in the summer during the cooling period. However, while the energy charges change only slightly between the seasons, the demand charges dominate the price of electricity.

Seattle. The local public utility classifies the building under investigation as a Large General Service customer to which Schedule 38 of the Electric Rate provision applies. The rates are detailed in Table 2.

Table 2. Seattle Energy Rates (Schedule 38)

	Summer (Apr.-Nov.)	Winter (Dec.-Mar.)
Energy charge:		
peak	3.13 c/kWh	3.86 c/kWh
off-peak	2.29 c/kWh	2.29 c/kWh
Energy demand:		
peak	0.59 \$/kW	1.16 \$/kW
off-peak	-	-
peak:	Monday-Friday	7am - 10pm
off-peak:	Monday-Friday Weekends	10pm - 7am Weekends

San Diego. The appropriate schedule for the San Diego utility is Schedule AL-TOU for General Service - Large. The rates are detailed in Table 3.

Table 3. San Diego Schedule AL-TOU (General Service-Large)

	Summer (May-Sep.)	Winter (Oct.-Apr.)
Energy charge:		
peak	7.95 c/kWh	7.13 c/kWh
semi-peak	5.23 c/kWh	4.46 c/kWh
off-peak	3.89 c/kWh	3.77 c/kWh
Energy demand:		
peak	17.54 \$/kW	4.08 \$/kW
semi-peak	-	-
off-peak	-	-
peak	11am - 6pm week	5pm - 8pm week
semi-peak	6am - 11am week 6pm - 10pm week	6am - 5pm week 8pm - 10pm week
off-peak	10pm - 6am week weekends	10pm - 6am week weekends

Optimization Formulation and Solution Methodology

The optimal operating strategy for cooling that incorporates ice storage capacity involves the minimization of the operating cost for a time period. The period in this investigation is chosen to be one week. Mathematically, the problem can be expressed as:

Minimize

$$L = \int_{0}^{1 \text{ week}} E_u(t) P(t) dt \quad (1)$$

subject to the system constraints that the chiller be able to handle the maximum load:

maximal chiller capacity:

$$\dot{Q}_{chiller}(t) \leq \dot{Q}_{chmax}(t) \quad (2)$$

A Dynamic Programming technique is chosen to solve this optimal solution problem. This method requires a recasting of the minimization problem from its continuous form into a time discrete format (Bellman 1957). The objective function of Equation (1) would then be expressed in hourly equidistant steps as:

$$L = \sum_{k=1}^K (E_u(k) P(k) \Delta t) \quad K=1, \dots, 168 \quad (3)$$

with 168 hours per week. The corresponding discrete constraints are:

$$\dot{Q}_{chiller}(k) \leq \dot{Q}_{chmax}(k) \quad (4)$$

with L = the total operating costs for one week,

$E_u(t)$ = unit energy charge,

P = electrical power,

$\dot{Q}_{chiller}$ = chiller heat flux.

\dot{Q}_{chmax} = max. chiller heat flux.

The decision variable in the optimization is the dimensionless level of storage $X(k)$ as a function of the discrete time step k . $X(k)$ varies from $X(k)=1$ (full storage) to $X(k)=0$ (empty storage). The time increment is one hour. Any negative change of storage level ($\Delta X = (X(k+1)-X(k)) < 0$) provides cooling from the storage. The cooling rate is then:

$$\dot{Q}_{disch} = -Cap \Delta X / \Delta t. \quad (5)$$

A positive change of ΔX requires a cooling of the storage of:

$$\dot{Q}_{charge} = Cap \Delta X / \Delta t. \quad (6)$$

with Cap = total capacity of the ice storage in MWh.

With a prescribed cooling load profile for the whole week and defined storage and chiller capacities, the Dynamic Programming method searches for the least cost strategy that satisfies the prescribed loads. The loads can either be met by direct chiller cooling or by chiller cooling supplemented with ice storage discharge, or solely by ice storage. The costs associated with an hourly storage charge of ΔX can be evaluated using a simple quadratic relation for the chiller performance:

$$P_{el} = b_2 (\dot{Q}_{charge})^2 + b_1 \dot{Q}_{charge} + b_0 \quad (7)$$

or non-dimensionalized as:

$$\frac{P_{el}}{P_{elmax}} = a_2 \left(\frac{\dot{Q}_{charge}}{\dot{Q}_{charge max}} \right)^2 + a_1 \left(\frac{\dot{Q}_{charge}}{\dot{Q}_{charge max}} \right) + a_0 \quad (8)$$

where the coefficients a_0 , a_1 , and a_2 were fitted for a reciprocating compressor chiller. Multiplying the electrical energy ($P_{el} \Delta t$) for one hour with the energy unit price $E_u(k)$ of that hour yields the operating cost for an incremental charge of ΔX .

The demand charge is determined for the investigated time period based solely on the highest electric energy use of the cooling system during the peak period. For any storage charge increment ΔX , the electric power $P_{el}(k)$ and the corresponding demand charge are evaluated and compared with the current demand charge up to that time stage. If this storage increment ΔX would cause a demand charge higher than the current charge, the additional charge is then added to the operating costs.

The optimization is performed for each representative week of the three seasons. The resulting minimal operating cost of that week is then multiplied by the number of weeks in that period to represent the minimal seasonal operating cost. The total annual cost is the sum of all minimal seasonal operating costs, the annualize capital cost for chiller and storage, and the maintenance cost.

Discussion of Results

The Seattle and San Diego locations represent two very different cases: a rather moderate summer temperature for Seattle and very low energy rates; and San Diego with relatively high temperatures and solar gains in the summer and high energy demand charges. The optimal chiller and storage size are given in Table 4:

Table 4. Optimal Chiller/Storage Size for Seattle and San Diego

	<u>Seattle</u>	<u>San Diego</u>
Chiller rating	1.1 MW	1.5 MW
Storage capacity	4 MWh	6 MWh
Total cost per year	\$36,897	\$96,906

The optimal control solution for the optimal chiller/storage combination for the Seattle climate is shown in Figure 5. The solution exhibits a partial storage behavior with load leveling characteristics to reduce, but not entirely off-set, direct chiller cooling during weekdays peak periods. The off-peak night period used for storage charging is constrained by the cooling load requirements to hours during which the cooling demand is zero. This system constraint stems from the assumption that only one chiller is used that can be operated either in the direct cooling mode or in the low temperature mode for storage recharging. To utilize the full 10 hour off-peak duration over night in the Seattle case, a dedicated stand-alone chiller would be required that could operated independently of the cooling load demand. The optimal solution in Figure 5 indicates full charging and discharging cycling during high load weekdays as the least cost operating protocol. If the storage capacities exceeds 8MWh, the empty-full storage cycle is extended to one full week where full storage is reached Monday morning and complete depletion after Saturday's last cooling hour. In this case, the daily cycling would have an overall downward trend, recharging every night to a level less than that of the previous day.

Due to the periodicity of the minimization problem of one week, different initial and final condition for the storage level had to be investigated to find the global solution. It

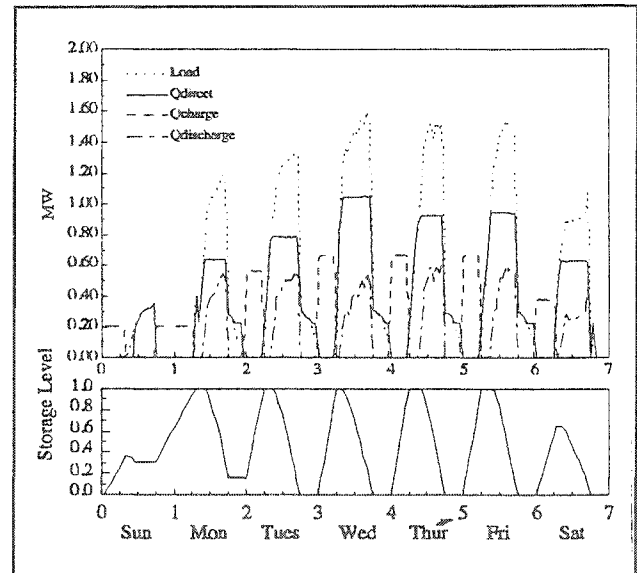


Figure 5. Optimal Control Solution of Summer Week Load Curve for Seattle

was found that the "empty storage condition" ($X=0$) as initial and final optimization condition yield the overall minimum. The sensitivity of the objective function to the initial condition was evaluated to be low. Changing the conditions from $X=0$ to $X=1.0$ changed the objective value by about 1% for the Seattle case with the optimal chiller/storage combination.

Figure 6 shows the optimal solution for the minimal storage and chiller capacities that meets the cooling loads throughout the year. The solid lines represent the edge of feasibility meaning that only on and above this line are the chiller and storage capacities sufficient to meet the cooling loads. Any lowering of the chiller capacity while keeping the storage capacity fixed would yield an infeasible solution and, thus, violate the cooling demand constraints.

Postulating that only those designs of interest are those that satisfy the cooling demands at all times limits the design space to be at or above the feasibility line.

Considering the zero storage option on the feasibility line, the load characteristics for Seattle require 1.6 MW (455 tons) while for San Diego weather 2.3 MW (654 tons) chiller capacity is necessary. The feasibility line further indicates a storage capacity limit at which any further increase of storage capacity would no longer contribute to a reduction of the chiller rating. For the Seattle case, this point is reached at 4 MWh (1137 ton-hours) storage capacity. For San Diego, this point is shifted to 8 MWh (2274 ton-hours). At these points, the time intervals for

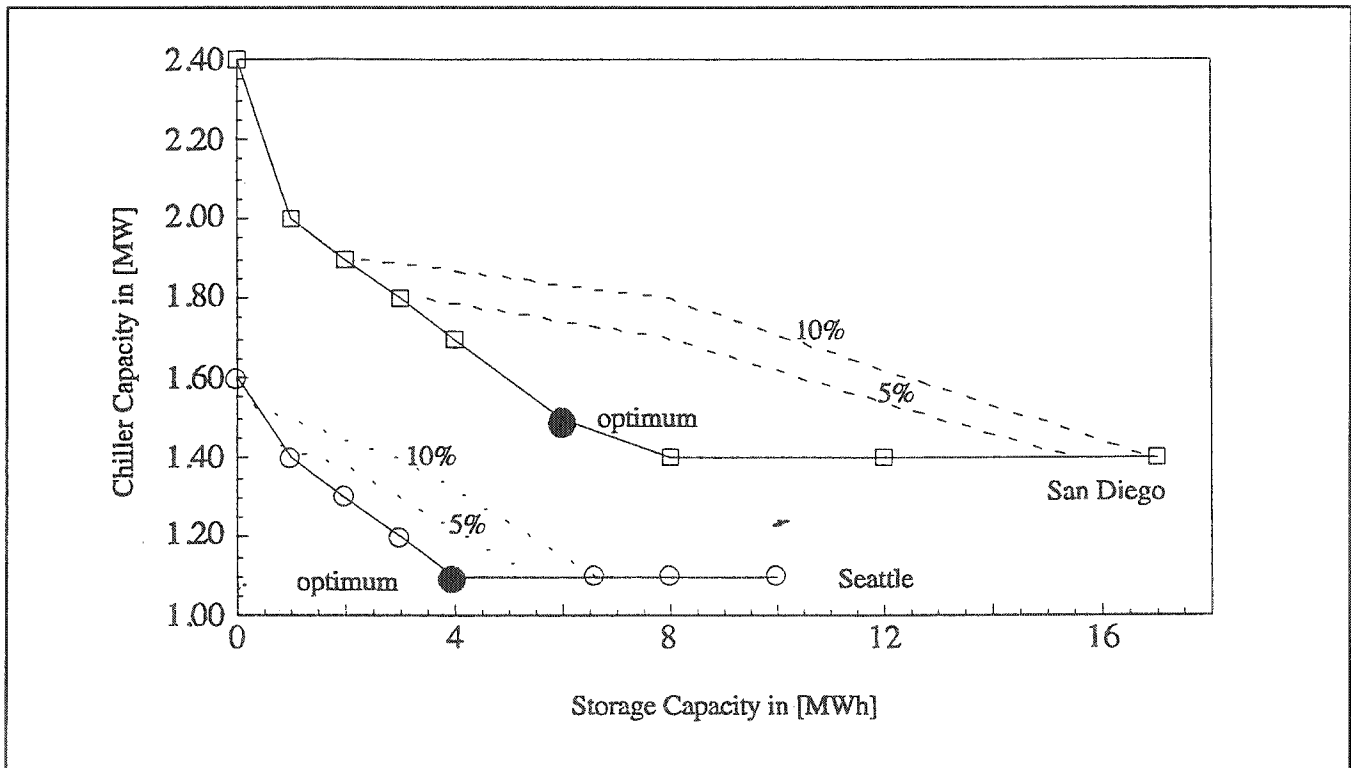


Figure 6. Optimum Chiller and Storage Size for Seattle and San Diego

storage recharging are the limiting factor for further utilizing additional storage. Since the chiller can only operate in one mode at a time, the night periods are not sufficient to allow more ice to be created. If storage capacity beyond this limit is desired, a specially designated chiller for ice production should be used. This additional chiller could then operate independently from the main chiller.

As mentioned before, the feasibility represents a design solution based solely on the cooling loads of the TMY weather data. It is unlikely to be used as a practical solution because of its lack of cooling reserve. To allow for systems reserves, the design engineer would be likely to choose a chiller-storage configuration above the feasibility line. The additional cost for selecting a non-optimal configuration are shown in Figure 6 and expressed in terms of sub-optimality defined as the fraction of the increased cost above minimum annual costs divided by the minimum annual cost.

The utility of Figure 6 for the design process comes into effect when the design engineer investigates an array of design options. The knowledge about the cost relations with respect to the chillers and storage capacities for a given climatic scenario provides insight into the economics of each design variant.

The results for the three short time temperature scenarios are shown for the Seattle case in Figure 7. The three upper curves represent the relationship between chiller and storage capacity which minimizes the weekly costs for each individual temperature scenario. Any combination of chiller and storage rating along the feasibility lines would satisfy the cooling loads of the respective temperature scenarios. The percentage sub-optimality with respect to the optimal yearly cost associated with the TMY base are shown for selected chiller/storage combinations. For the constant mean temperature, for instance, the sub-optimality case was evaluated at discrete points on the line and ranges between 48% and 52% sub-optimality. In other words, if the designer opts for a system that would satisfy the load requirement for the assumed constant temperature heat wave scenario in Seattle, the design choices would necessitate costs between 48% and 52% over the optimal cost assuming the weather for the whole year would be according to the TMY weather data. The significance of this sensitivity analysis lies in the correlation of temperature scenario assumptions and the technical feasibility with its associated costs. Given these temperature scenario dependent feasibility curves, the builder and designer can make trade-off choices between sufficient cooling during rare but not totally impossible heat wave periods and required costs.

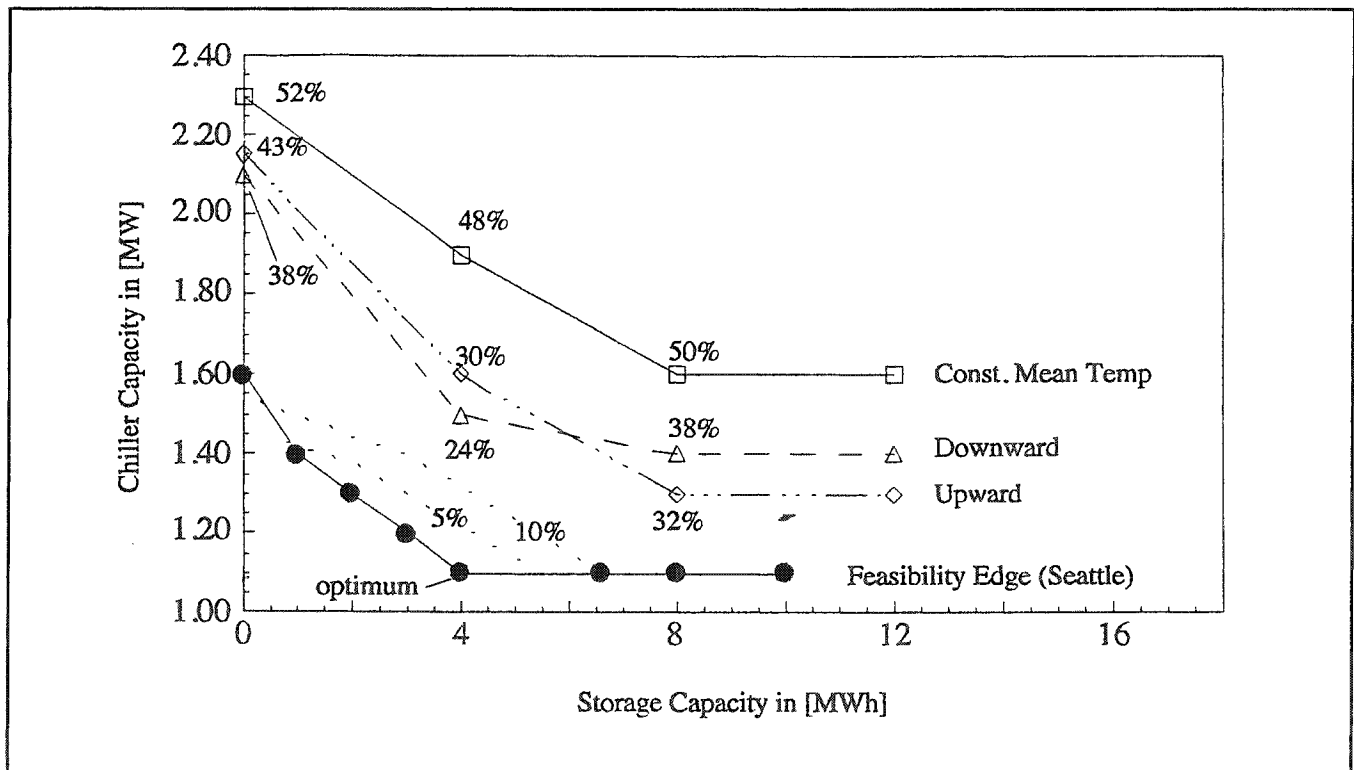


Figure 7. Temperature Scenarios for Seattle

Another sensitivity analysis was performed by analyzing the ratio of energy charges of off-peak to peak period charges. Figure 8 illustrates the change of the optimal storage capacity with changing summer electric rate ratios. From Figure 8 it is apparent that the optimal storage selection is insensitive to changes of the energy rate ratio for the Seattle case and highly sensitive for San Diego. An extreme value of 10 times the current peak rate was used to study changes in the optimal storage size. As the rate ratio approaches unity the cost savings are attributable only to the down-sizing of the chiller. Cost-savings of operating costs are diminished due to lack of sensitivity to load shifting to off-peak periods.

With regards to the overall annualized cost, both cases show distinct differences (see Figure 9). The x-axis represents the capacity ratio of storage to that of the chiller. The normalizing chiller rating is the minimal rating for a given storage capacity for which the cooling loads are still satisfied. The resulting units of this ratio is hours which can be interpreted as a time parameter specific to the building cooling characteristic as reflected in the load curve. For the optimal chiller and storage capacities values the ratio is 3.6 and 4.0 for Seattle and San Diego, respectively. Although both cases investigated

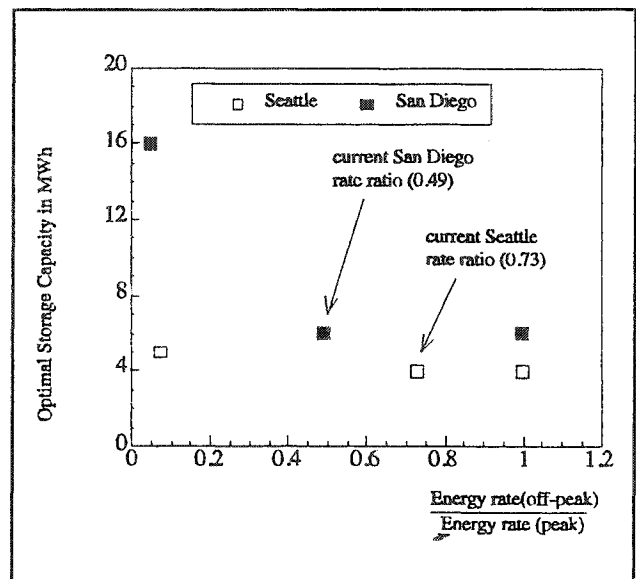


Figure 8. Energy Rate Ratio Sensitivity for Seattle and San Diego

represent two extreme differences in the climate and energy rates, the ratio for optimal sizing are very similar in magnitude for both cases.

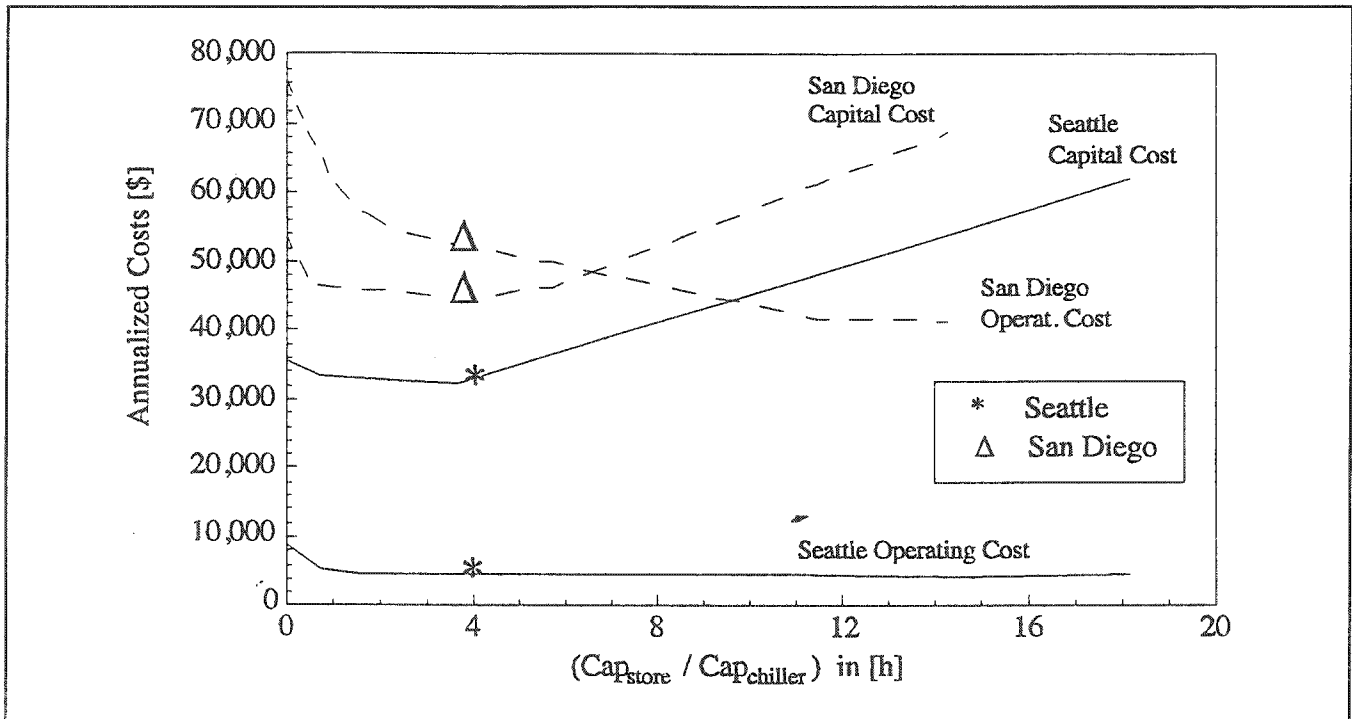


Figure 9. Annualized Capital and Operating Costs for Seattle and San Diego

More research is required to study the capacity ratio for the optimal solution to verify if this ratio is mainly a function of the building cooling characteristic and relatively independent of the energy rates. If this were true, then design for different climatic zones would be simplified.

Conclusions

The optimal selections of storage and chiller capacity were performed for two climatically different US locations, Seattle and San Diego. The optimal selection of chiller and storage was evaluated by solving the optimal control problem of a given storage and chiller, thus identifying the feasibility line.

The optimal selection for the Seattle case was predominantly determined by the capital costs. Operating costs played an insignificant part. For the San Diego case, both the capital and operating costs are approximately equal. The optimal storage/chiller selection for Seattle was $CAP_{chiller} = 1.1$ MW (312 ton) and $CAP_{store} = 4$ MWh (1137 ton-hours) and for San Diego $CAP_{chiller} = 1.5$ MW (426 ton) and $CAP_{store} = 6$ MWh (1705 ton-hours).

The outdoor temperature sensitivity for the Seattle case indicated that if a system is to meet the cooling loads during the one week high temperature scenario an

additional 48% to 52% of the total optimal annualized costs will be incurred. The electric rate ratio sensitivity analysis showed very little effect on the optimal selection for the Seattle case. But in the San Diego case, the results display a high degree of sensitivity.

The ratio of storage versus chiller capacity for the optimal sizing was evaluated as a value close to 4.0 hours in both cases, despite major differences in climate and energy rates between the two locations. This suggests that the critical parameter is the time constant of the building. If this time is less than the charging time available, then an optimal control is possible. If not, then we speculate that an optimal control may not be possible. Further research must be conducted to study this behavior for a wide range of energy rate difference and different climates.

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