

**BUILDING AS A DYNAMIC CALORIMETER:
DETERMINATION OF HEATING SYSTEM EFFICIENCY**

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ABSTRACT

A building can be used as a dynamic calorimeter to determine an unknown, complex energy flow that is difficult or impossible to measure directly. Examples of such intractable processes include delivered energy from an HVAC system and solar gains. The basic idea is to first construct and calibrate a building model that allows inference of all major energy flows in the building (excluding the one unknown complex flux) from a few simple measurements. One then admits the unknown flow and relies on the whole building energy balance to determine the unknown heat flow. In this article, whole building calorimetry is used to determine the efficiency of a residential forced air gas heating system, before and after retrofitting the system.

The building model must first be chosen as appropriate to relate heat flows to measured driving functions. Building parameters which are input to the model must then be determined. The data channels for obtaining the parameters for heating problems typically include a few temperatures, incident solar radiation, electric power into the building, and wind speed. It is necessary to use limited control of internal temperatures and electricity gains during the short term tests to minimize errors in parameter extraction. In other words, it is essential to follow a carefully specified test protocol.

After this calibration, the heating system to the house was turned on and observed for several days. Heating system efficiency is obtained by integrating the heat flows over some interval and dividing by the measured total gas input. This efficiency is the net result of combustion losses as well as distribution system losses to the unconditioned basement. For the furnace tested, initial efficiency was $52 \pm 2\%$. After the system was retrofitted (mainly increasing the fan flow rate and taping some duct leaks), the efficiency increased to $58 \pm 2\%$.

Building calorimetry has implications far beyond the measurement technique demonstrated in this paper. One related application of interest is on-line diagnosis of HVAC systems, based upon measurements typically available to an energy management system.

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INTRODUCTION

Monitored data on the thermal performance of a building contains a wealth of information that is useful for a variety of applications. In order to fully realize this potential, it is essential (1) to have a sound thermal model of the building, and (2) to acquire the appropriate data so that the model is useful for the particular application. As a time-integrated example, utility meter readings give useful information when combined with a degree-day model (Fels 1984). In this article, we concentrate on applications that require dynamic (hourly) data from a small number of data channels. Typically, in residences such data are acquired with a data logger through short-term tests and in commercial buildings with an Energy Management and Control System (EMCS).

The primary analytical method is "macrodynamics" (Subbarao 1985) - an hourly simulation whose inputs are a small number of aggregated parameters that are both (a) calculable from a detailed building description (when such a description is available), and (b) measurable from performance data from a small number of channels. When a detailed description (such as conductivities and thicknesses of wall layers) is not available, the measured macroparameters (such as the loss coefficient) implicitly contain the necessary information. Applications of macrodynamics include (a) long-term performance prediction from short-term tests, (b) diagnostics of space-conditioning equipment, (c) optimal control of HVAC systems, and (d) comparison of design vs. actual performance.

One application of macrodynamics involves using the building itself as a calorimeter in order to extract delivered thermal energy from a given source. In this method, the building is dynamically calibrated as a calorimeter, which means all dynamic energy flows in the building (including flows into storage) are obtainable from simple measurements. Then, an unknown heat flow is admitted, and the hourly values of the unknown flow determined from an energy balance. If the flow thus deduced is from a furnace, the net heat delivered to the space can be obtained, and, knowing gas input, net system efficiency determined. If the deduced flow is from solar radiation, actual solar gains can be determined. If the deduced flow is from HVAC systems in a commercial building, HVAC system performance can be monitored.

In order to test and refine these possibilities, measurement of net heat delivered by the heating system is highlighted in this article. Short-term tests were performed on an unoccupied residential building to obtain the thermal parameters which are inputs for a thermal model of the building. With the furnace operating, the actual heat input to the living space was deduced using the building as a calorimeter. From the measured gas input, the heating

system efficiency was obtained to be 51.5 (± 1.9)%. Furthermore, the furnace was retrofitted with a fan, with a capacity approximately double that of the original, and some leaks in the ducts taped. A measurement of the efficiency after these changes gave a value of 58.1 (± 1.9)%. This measurement of heating system efficiency is a combination of combustion efficiency as well as that of the distribution. Furnace efficiency was determined from separate tests to be about 82%.

The dynamic calibration method should be compared with the static calibration method. When the inside and outside temperatures are constant and when solar effects are small (such as at night time for a light mass building), the building loss coefficient obtained from electric coheating provides the necessary calibration to determine the furnace efficiency (SERI 1983). Dynamic calibration accounts for thermal mass effects and therefore it is not necessary to require that the inside and outside temperatures remain constant during the measurement period. Solar gains, and discharging of solar-charged mass are also accounted for. Thus the dynamic method is applicable to a wider class of buildings and under a broader range of experimental conditions. The dynamic method given here is similar to the equivalent thermal parameter method (Kusuda, 1971, Kimura, 1972, Sonderegger 1978). The main differences are (i) we define the building parameters differently and (ii) we determine the parameters from a specified short-term test protocol designed to elicit all the important building responses. A more detailed discussion is given later in this article.

In Section II, the theoretical considerations that lead to a dynamic model of the building are outlined. In Section III short-term tests that led to a determination of these parameters are given. These parameters essentially provide a dynamic calibration of the building as a calorimeter. In Section IV, furnace experiments are discussed. Section V contains an analysis of the data taken and the results of the analysis. Section VI contains a discussion of the method, limitations and further refinements. Section VII contains a summary and conclusions.

OUTLINE OF THE THEORY

In this section a brief outline of the theory behind macrodynamics is given. The specific macrodynamic model used here is referred to as BEVA (Building Element Vector Analysis). A detailed presentation is given in (Subbarao 1983) and (Subbarao 1984a,b).

For a massless, one-zone building at temperature $T_{in}(t)$ the energy balance equation can be written as

$$(L_o + L_b) T_{in}(t) - L_o T_{out}(t) - L_b T_b(t) - S_o I_{sun}(t) = Q_{int}(t) + Q_{aux}(t) \quad (1)$$

In Equation (1), solar gains are modeled through a simple equivalent clear aperture area times radiation intensity on a reference orientation.

In order to include mass effects, Equation (1) needs modifications. For a linear system, the necessary modifications lead to

$$\int_{-\infty}^{\tau} \delta t' [V(t-t') T_{in}(t') - W(t-t') T_{out}(t') - W_b(t-t') T_b(t') - S(t-t') I_{sun}(t')] dt' = Q_{int}(t) + Q_{aux}(t) \quad (2)$$

In Equation (2) we have assumed auxiliary and internal gains to be instantaneous. (If not, they require their own transfer functions.) Variability of infiltration with stack and wind effects can be readily incorporated by defining an effective internal gains which is the net gain after subtracting measured/modeled variable infiltration losses from normally-defined internal gains throughout this paper, we shall assume, unless otherwise stated, that the internal gains are in fact the effective internal gains as defined above. When the basement temperature is essentially a constant, $\int W_b(t-t') T_b(t') dt'$ can be replaced by $L_b T_b$.

In order for Equation (2) to be manageable, it is essential to parameterize the various transfer functions to provide an adequate representation with as few parameters as possible. The following forms serve that purpose for a wide class of buildings (Subbarao 1984a):

$$\begin{aligned} V(t-t') &= (L_o + L_b) \delta(t-t') + U_{in} \left[\delta(t-t') - \frac{1}{\tau_{in}} e^{-(t-t')/\tau_{in}} \right] \\ W(t-t') &= L_o \delta(t-t') - U_{out} \left[\delta(t-t') - \frac{1}{\tau_{out}} e^{-(t-t')/\tau_{out}} \right] \\ S(t-t') &= S_o \delta(t-t') - A_{sun} \left[\delta(t-t') - \frac{1}{\tau_{sun}} e^{-(t-t')/\tau_{sun}} \right] \\ W_b(t-t') &= L_b \delta(t-t') \end{aligned} \quad (3)$$

A justification for the above forms as well as for the relationship to Fourier transforms can be found in (Subbarao 1984a).

The heat balance Equation (2) can be used to determine the net flow due to an unknown term, if we know the rest of the terms. For example, if all flows except the solar term are known, the solar gains "seen" by the room air can be determined. In this article, we shall use Equation (2) to determine $Q_{aux}(t)$ knowing all the other terms.

The left hand side of Equation (2) can be calculated from a knowledge of the present and past values of the inside, outside and basement temperatures and solar intensity, provided we know the nine parameters (L_o , L_b , U_{in} , τ_{in} , U_{out} , τ_{out} , S_o , A_{sun} , and τ_{sun}). These parameters essentially provide a dynamic calibration of the building as a calorimeter. They allow a determination of the heat supplied to inside air by the heating system, which includes both furnace and the distribution system.

EXPERIMENTAL DETERMINATION OF THE BUILDING PARAMETERS

The building chosen for testing during June 1985 is a single family detached residence near Denver, CO. The house has two natural thermal zones, an open conditioned upstairs of floor area 115 m^2 (1240 ft^2), and an unfinished totally open unconditioned basement of area 103 m^2 (1110 ft^2). Temperature variation from room to room upstairs was typically less than 0.5°C , due to an open floor plan and use of two mixing fans. Insulated wood-frame wall and ceiling were nominally R12 and R32 respectively. The house has 15.5 m^2 (167 ft^2) gross glass area, with 11.8 m^2 (127 ft^2) on the south orientation. The heating auxiliary is a natural gas forced air system, rated at 100,000 Btu/hr. The air distribution ducts are uninsulated sheet metal. The house was unoccupied for these tests.

In the previous section, the definition of the building parameters and their role in providing a dynamic calibration of the building as a calorimeter was outlined. How does one obtain these parameters experimentally? In principle, one can obtain them from a regression by minimizing the sum of squares of differences between predicted and measured inside temperature (or auxiliary energy, or some other suitable quantity). In practice, if short-term data under normal building operation are used, it is highly unlikely that all important effects are adequately stressed. One will have to use either long-term data or perform short-term tests that properly stress all the important effects.

In (Subbarao 1985), the appropriate test conditions were incorporated into a short-term test protocol which depends on the building. For the building here, the test protocol was:

- 1) Data Channels: T_{in} , T_{out} , I_{sun} , Q_{int} , T_b , and v_w .
- 2) Internal Gains Profiles: Three periods, approximately 2 days each
 - a) During the first period, heat input from the electric heaters was chosen to maintain reasonable comfort conditions, if required.
 - b) During the second period, electric heat was introduced which was roughly the same as in the first period, with an additional input

$$Q_{prof,2} = \bar{Q}_2 + \Delta Q \sin [\omega (t+t_0)].$$

The period of the sinusoidal heat input was chosen to be 16 hours ($\omega = 2\pi/16$), and $\Delta Q = \bar{Q}_2$

- c) During the third period, electrical heat was introduced at constant rate Q_3 with $Q_3 \sim 2 * \bar{Q}_2$
- 3) Additional One-time Measurements:

A few representative tracer gas measurements were done to determine infiltration model coefficients. Using quadrature addition of wind and stack effect, air changes per hour (a) is given by:

$$a = [b \cdot \Delta T + c \cdot v^2]^{\frac{1}{2}}$$

where b and c are the model coefficients. The ratio of b/c was fixed at values recommended in (Reinhold 1983).

The above test led to the building parameters listed in Table I. Details of the experiment and the parameter determination are given in (Subbarao 1985). A simple transformation is required to derive the values listed in Table I from values given in the reference.

HEATING SYSTEM MEASUREMENTS

In this section furnace measurements are described. The analysis of the data leading to a determination of the heat delivered to the living space is given in the next section.

The experiments were conducted on the Maplewood residence in June, 1985. In order to get any substantial furnace on-time, the thermostat was set at about 85°F. The furnace came on generally during early morning hours. Only data for those hours with significant furnace activity as well as for an hour or two following the shut-off of the furnace (to pick up any time delays introduced by mass effects in the furnace and distribution system) are of interest in the present context. The data for four such early morning periods are plotted in Figure 1. The top portion of the figure is a plot of the inside temperature, the outside temperature, the basement temperature, global horizontal solar flux as well as effective internal gains. Negative values of the effective internal gains occur when infiltration losses exceed incidental electrical gains. The bottom portion of the figure contains a plot of the measured hourly gas input to the furnace. Also plotted are the calculated hourly heat input to the living space. The analysis leading up to this latter curve are described in the next section.

Furnace efficiency was measured from a separate test as follows. With the burners of the furnace off, but the blower fan running, known electrical resistance heat was introduced in the duct. R-19 insulation was wrapped around this portion of the duct, extending between the positions of a nine-element differential thermopile. By measuring the temperature rise across the heater (and making the appropriate small correction due to losses to the surroundings) the air flow rate was determined to be about 670 cfm. Assuming

TABLE I. Building Parameters from Short-Term Tests

$L_o = 298$	Btu/hr°F	$S_o = 54.9$	ft ²
$L_B = 307$	Btu/hr°F	$\tau_{sun} = 3.25$	hrs
$\tau_{in} = 1.06$	hrs	$A_{sun} = 37.6$	ft ²
$U_{in} = 3368$	Btu/hr°F	$b = 1.86 \times 10^{-3}$	°F ⁻¹
$\tau_{out} = 8.69$	hrs	$c = 0.70 \times 10^{-3}$	hr ² /miles ²
$U_{out} = 274$	Btu/hr°F		

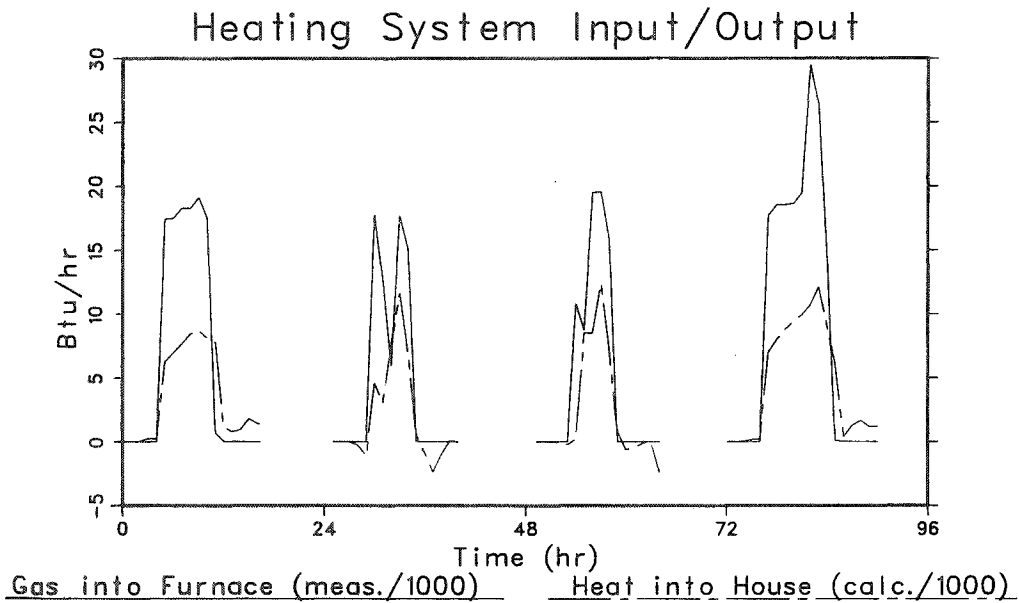
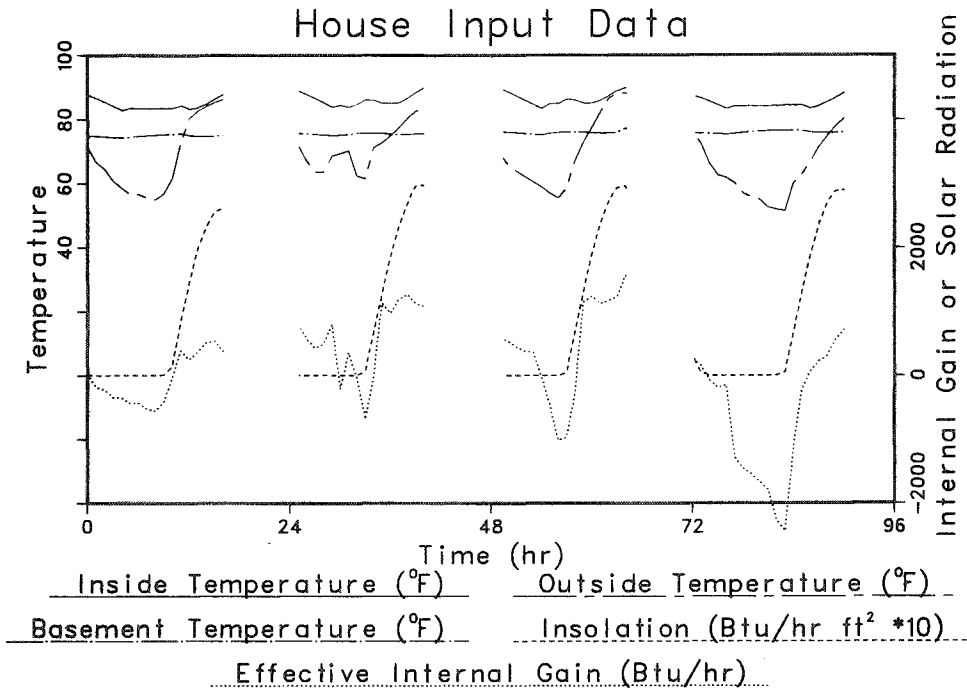


Figure 1 Data plots for the Maplewood Residence. Figure 1.a gives the data channels necessary to predict the heat into the house, given the house parameters in Table I. Figure 1.b shows the measured fuel input to the furnace and the calculated heat delivered to the house. These data cover four days before the furnace was retrofitted. Data after the retrofit were qualitatively similar and are not shown.

the same flowrate to persist when the electric heater is turned off and the furnace burners turned on, combustion efficiency can be determined by measuring the temperature rise across the furnace and the gas flowrate. This gave about 82% for the steady state combustion efficiency.

In order to test the building-as-a-calorimeter technique further (especially for retrofit applications), the following additional experiments were done. The furnace fan was replaced with a fan with approximately double the capacity (~1170 cfm), and some leaks in the ducts were sealed. Data for three early morning periods were taken in a manner similar to pre-retrofit. The resulting increase in the heat delivered to living space is analyzed in the next section.

ANALYSIS OF DATA FROM FURNACE MEASUREMENTS

Experiments and results for the determination of the thermal parameters for the Maplewood residence were described above. These parameters provide a calibration of the building as a dynamic calorimeter. The heat $Q_{aux}(t)$ actually delivered to the living space of the house (excluding the basement) can now be calculated. In order to do this calculation, we need to discretize Equation 2 to accept average hourly values. Also, we need to do the necessary manipulations so that the infinite integrals are eliminated in favor of past values of heat flows. These details are quite straightforward. The resulting $Q_{aux}(t)$ is plotted in the lower part of the Figure 1. Also plotted is the measured gas input to the furnace as a function of time. As expected, there are no major time delays introduced by the distribution system.

In order to proceed further, we need a suitable model of the furnace. Quite detailed models of the furnace are available (Gable 1977). These models typically require finer resolution than an hour. In order to keep things simple, in this article we shall use the simplest model of the furnace, namely a constant efficiency model.

During each of the four periods in the figure, we shall total the gas input as well as the calculated delivered heat output. The ratio of the latter to the former is the average efficiency over the period. Table II summarizes the average efficiency for the four periods in the graph for three different choices of starting hours. The variation from one period to another can be due to inadequate modeling of the furnace. Cycling characteristics, furnace

TABLE II. Furnace Efficiency (%) Before Retrofit

	PERIOD 1	PERIOD 2	PERIOD 3	PERIOD 4
Avg. Gas Input (BTU/hr)	15540	11510	10640	18200
Starting Point A	49.6	49.7	50.4	49.5
Starting Point B	44.1	58.0	53.0	46.4
Starting Point C	51.0	72.6	54.4	48.8

mass effects, etc., have not been modeled in detail. The apparent efficiency can therefore vary from period to period. On the other hand, the sensitivity for a given period to the choice of the initial hour is primarily due to limitations of building modeling and/or errors in data. The sensitivity is exacerbated if the furnace on times are rather small (as was the case with our summer measurements.) We believe that the average efficiency figures are reliable. (The initialization issue discussed here should not be confused with the initialization issue encountered in design simulations where either the temperatures of nodes or the past values of inside temperature are usually set to some convenient values, and the first few hours or days are simulated in a manner so as to dilute the effect of the choice of initial conditions.) The mean of the efficiency numbers for each period and starting condition, weighted by the total gas input for each period, is $51.5 \pm 2\%$. The quoted uncertainty is the standard deviation of the mean, treating each of the twelve measurements as an independent measurement. Systematic errors due to modeling, sensor calibration, etc., are not included.

The combustion efficiency as measured in Section IV was 82%. Therefore, of the heat output by the furnace, about 63% is delivered to the living area and the rest - about 37% - is "lost" in the distribution system, i.e., ends up in the basement. This approach using the building as a calorimeter should be compared with direct measurement of heat delivered to living space using hoods over registers (Andrews 1985). Such measurements show, as expected, considerable variation from building to building; our results are within the range seen in such measurements.

The data from the three post-retrofit periods were analyzed in the same manner as the pre-retrofit data. The results are given in Table III. The three different rows for each period are, as before, from three different starting points for the hourly simulation. The weighted mean of these numbers gives a post-retrofit furnace efficiency of $58.1 \pm 2\%$. The distribution system losses were reduced from about 37% to about 29%. The more powerful fan reduces the temperature of the air flowing through the ducts and thus decreases duct losses; sealing duct leaks obviously decreases losses. (The more powerful fan consumes higher electrical energy which also ends up in the air stream. The fan energy is subtracted in both pre- and post-retrofit analysis in deriving net delivered energy).

TABLE III. Heating System Efficiency (%) After Retrofit

	PERIOD 1	PERIOD 2	PERIOD 3
Gas Input (BTU/hr)	12920	15720	13660
Starting Point A	50.2	70.1	56.0
Starting Point B	61.2	57.2	54.9
Starting Point C	62.7	53.2	56.8

Most of the energy "lost" in the distribution actually ends up in the basement, and tends to elevate the basement temperature. As a result, heat loss from the living space to the basement is reduced. This secondary effect could be treated as an upward readjustment of the efficiency numbers given above. In order to estimate this effect, we need a detailed model of the basement including ground heat flows; we have not done this.

DISCUSSION

In the previous section we showed how the building can be used as a dynamically calibrated calorimeter, and thereby determined the hourly heat input from the furnace to the living space. When coupled with a model of the HVAC system, the building-as-a-calorimeter can provide a measurement-based characterization as well as diagnostics of the HVAC system. For the furnace, we used a simple single efficiency model to determine the pre-retrofit efficiency to be $51.5 \pm 2\%$ and post-retrofit efficiency to be $58.1 \pm 2\%$. These results provide evidence of the usefulness of the approach. In this section we discuss some of the sources of errors and possible improvements.

There are two sources of limitations - one in the calibration of the building and another in the modeling of the HVAC system. If we have a good calibration of the building, the calculated hourly heat flows such as those in the bottom part of Fig. 1 will be reliable. What we do with these heat flows depends on HVAC modeling. For the furnace, we could use additional parameters to account for cycling, etc. One should be careful not to have too many parameters, for then the parameters are likely to be ill-determined.

The adequacy of calibration of the building can be easily tested by calculating the apparent heat delivered by the furnace during periods when it is known to be zero. This flow is typically in the (\pm) hundreds of Btu range. This should be compared with the typical magnitudes of the various paths of heat flow (such as from the living space to the basement, to outside, solar gains, etc.) which are in the thousands of Btu range. Further improvements in the building calibration can be expected by including the effects of sky radiation, better solar modeling for multiple orientation effects, etc. Such improvements are being pursued.

Similarities and differences between the present macrodynamic method and the equivalent thermal parameter method referred to in the introduction will now be outlined. The present method identifies the major driving functions and parameterizes them using the BEVA framework. All the building parameters are thus calculable in a simple manner from a building description. It is quite possible that a simpler circuit containing fewer parameters, such as those in (Kusuda 1972), (Sonderegger 1978) gives essentially as good a fit, especially for short-term data. In this case, however, some of the circuit parameters become complicated functions of building details and the driving functions, and essentially are quite intractable. The second major difference is that the BEVA model parameters can be obtained from short-term tests designed to elicit these parameters. This ameliorates the critical problem commonly encountered in building parameters estimation, namely, that the parameters are often so poorly determined that different periods give quite different

estimates. This difficulty would occur with the present method also if the building were not subjected to the testing protocol discussed above.

SUMMARY AND CONCLUSIONS

The building can be used as a dynamically calibrated calorimeter in order to measure otherwise unknown heat flows. The calibration can be achieved by: (a) identifying all the driving functions - such as the inside, outside, and basement temperatures, solar radiation, wind speed, and internal gains- that determine heat flows to and from the building; (b) properly parameterizing the building response to these driving functions; and (c) performing short-term tests that elicit these responses and thereby obtaining the parameters. Macro dynamics provides the proper theoretical and practical foundations for accomplishing this calibration.

Having dynamically calibrated the building, one can determine unknown heat flows by an energy balance. By working with data from a residential building, it was shown how heat delivered to living space can be determined. A simple constant-efficiency model of the furnace led to a net efficiency of $51.5 \pm 1.9\%$. When the furnace was retrofitted with a blower of a higher capacity and some duct leaks sealed, the efficiency increased to $58.1 \pm 1.9\%$.

By generalizing the above application, it should be possible to perform real-time diagnostics of HVAC systems in commercial buildings using data from EMCS perhaps with a few additional channels. Dynamic calorimetry also allows a determination of heat gains from solar radiation, thereby allowing experimental evaluation of energy efficient designs. Net energy delivered by "exotic" systems, such as a phase change storage wall or a liquid diode collector, can be determined if the rest of the building is used as a dynamically calibrated calorimeter.

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NOMENCLATURESymbols

a	Air changes per hour
A	Area
b	Constant in infiltration relation, multiplying temperature difference
c	Constant in infiltration relation, multiplying wind velocity
δ	Delta function
Δ	Difference
I	Solar irradiation
L	Steady-state thermal coupling
Q	Quantity of energy, or energy per unit/time
S	Transfer function for solar radiation, or solar area
T	Temperature
τ	Time constant
t	Time
U	Conductance
v	Wind Velocity
V	Transfer function for indoor temperature
W	Transfer function for outdoor temperature
ω	Angular frequency

Subscripts

aux	Auxiliary
b	Basement
in	Inside
int	Internal gain
o	Outside
out	Outside
sun	Solar radiation

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