

A MICROCOMPUTER ALGORITHM FOR DEGREE-DAYS TO AN ARBITRARY BASE

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ABSTRACT

For energy-efficient buildings, many analysts need estimates of annual heating degree-days measured from a base temperature different than the conventional 65 °F. This paper presents an algorithm to estimate annual heating degree-days (AHDD) from an arbitrary base temperature. The algorithm may be used with a pocket calculator, or may be incorporated in microcomputer modeling programs that analyze energy use in buildings. It is based on a physical climate model with two parameters: the annual average temperature, and the average temperature of the coldest month.

Algorithm results are compared to degree-day data published by the Los Alamos National Laboratory (LANL) for every state in the United States and for four base temperatures. The average difference between algorithm estimates and LANL data is 15 AHDD, with a standard deviation of 168 AHDD. This standard deviation is about half the standard deviation of year-to-year fluctuations in AHDD in most parts of the United States.

The correlation coefficient between algorithm data and the comparison data is 0.998 over the range 0 - 20,000 AHDD, and 0 - 6000 annual cooling degree-days.

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INTRODUCTION

As buildings become more energy efficient, there is increased interest in making accurate estimates of seasonal heating costs. Accurate heating cost estimates are important to architects and their clients, to utility load forecasting departments, and to officials who wish to develop "energy labels" or "home energy rating systems".

For energy-efficient buildings, many analysts need estimates of annual heating degree-days (AHDD) measured from a base temperature different from the conventional 65 °F* to describe the heating load that a building faces. The appropriate base temperature is a feature of a building's design, and its lighting and equipment--not the external climate. The more energy efficient a building is, the lower the appropriate base temperature becomes.

This paper presents an algorithm to estimate AHDD to an arbitrary base temperature, and compares its results with published data. The algorithm:

1. Is based on a two-parameter physical climate model that requires specification of the annual average temperature and the average January temperature for a given site--data available from any almanac;
2. Is calculable on a microcomputer or on any pocket calculator having inverse trigonometric functions. It may be incorporated as a subroutine in larger computer analyses;
3. May be sufficiently accurate for many design or estimation applications. It reproduces published data within 15 AHDD on the average, and with a standard deviation that is about half that of year-to-year fluctuations of AHDD in most parts of the United States;
4. Is usable over a very wide range of climates (0 - 20,000 AHDD and 0 - 6000 annual cooling degree-days);

* In this paper all temperatures are on the Fahrenheit scale to facilitate comparisons with published tabular data also expressed in Fahrenheit units.

5. May be useful where relative comparisons are needed--like the EPA mileage figures for automobiles--i.e., to estimate the change in the heating load of a specific building when design parameters are changed.

As the energy efficiency of the Nation's building stock slowly improves, the cumulative effects of degree-day inaccuracy may become large. Very roughly, a change of 1 °F in the base temperature corresponds to a change of about 200 AHDD in the heating load of a single family dwelling. Extrapolated to the residential building stock of the United States, this is equivalent to about 24 billion Kwh, or the output of five 1000 Mw power plants with a capacity factor of 0.6.

THE CLIMATE MODEL

Because the earth's axis is tilted, the annual temperature in most parts of the world, when smoothed by averaging, can be expected to vary sinusoidally.

The model we use to describe annual temperature fluctuations is a sine curve, centered on the annual average temperature of a site in question. The amplitude and phase of the sine curve is such that the January "histogram" is just enclosed, as shown in Figure 1.

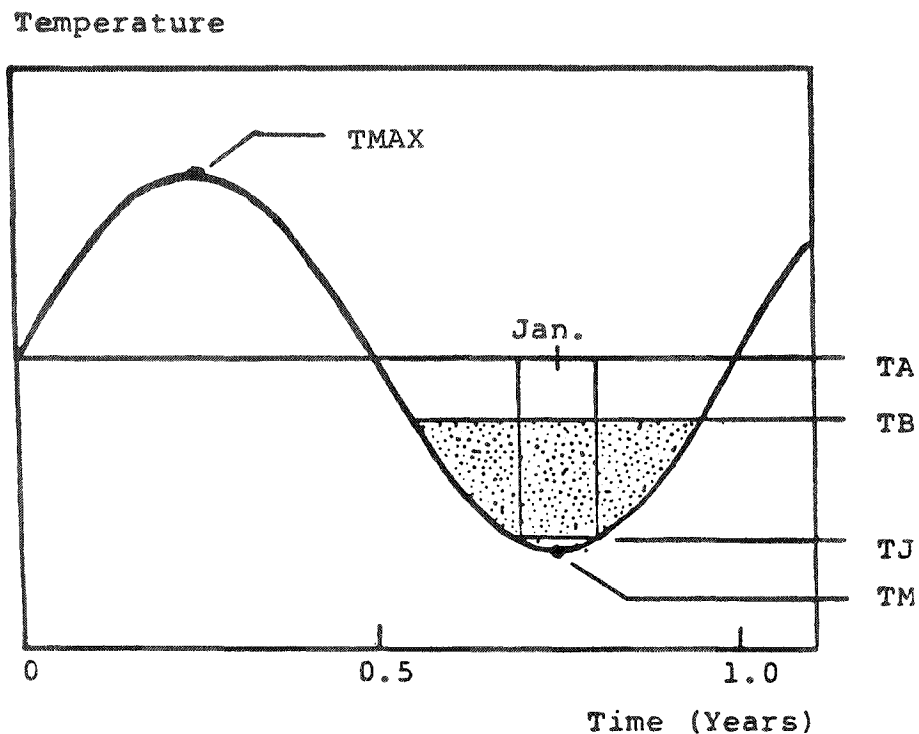


Figure 1. Climate model.

In this sine wave model, AHDD to the base temperature TB corresponds to the stippled area in Figure 1--below the horizontal line T = TB and above the sine curve.

THE ANALYTIC APPROXIMATION

The stippled area in Figure 1 can be found by integration. DD(TB), representing the degree-day function for the arbitrary base temperature TB, is given by:

$$DD(TB) = \frac{Y}{\pi} (TA - TM) [\sqrt{1 - x^2} - x \cos^{-1} x]$$

Where $Y = 365.26$ (days per sidereal year)

$$\pi = \text{Pi} = 3.14159$$

TA = annual average temperature (dry bulb)
(Fahrenheit, Celsius, Kelvin ...)

TJ = average January temperature

TM = kTJ - (k-1)TA; minimum of sine curve
temperature

k = 1.036; numerical constant to fit sine curve
to just enclose January "histogram"

$$x = (TA - TB) / (TA - TM) \text{ (radians)}$$

And for the two extreme cases:

1. DD(TB) = 0 for TB \leq TM (no heating required), and
2. DD(TB) = Y(TB - TA) for TB \geq TMAX (year round heating)
Where TMAX = TA + (TA - TM) = 2TA - TM.

Figure 1 shows the variables of the algorithm graphically.

As the base temperature, TB, nears the minimum annual temperature, TM, AHDD approaches zero. Near TB = TM, diurnal temperature fluctuations, not modeled by the sine wave, become dominant. In this regime, heating costs also approach zero, so accurate estimates are unnecessary.

This method of estimating AHDD differs from that of Thom [4]. It is based on a physical model rather than a statistical model. It requires only two data points to model the yearly climate rather than the average temperature and its standard deviation for each of 12 months.

An interactive BASIC program (DDAY.BAS) to perform these calculations may be downloaded from Directory "basic" of the U.S. Department of Energy's Electronic Bulletin Board. (1200 N 8 1, (301) 353-4892 or FTS 233-4892).

COMPARISON WITH DATA

Annual Heating Degree-Days

Using data from the National Oceanic and Atmospheric Administration (NOAA) [3], Los Alamos National Laboratory (LANL) has published [2] annual heating degree-day estimates (based on daily mean temperature data) for base temperatures of 50, 55, 60, and 65 °F, for 209 U.S. cities. We selected 55 of those cities, one for each of the 48 contiguous states, and more than one for several large states.

For the 220 combinations of city and base temperature, the analytic approximation was an average of 15 AHDD lower than the LANL data. The standard deviation of the difference between LANL data and the analytic approximation is 168 AHDD. By way of comparison, the average standard deviation of year-to-year fluctuations in AHDD for a sample of 24 U.S. cities (over the 20-year period 1958-1978) was 340 AHDD [1]. Summary data for individual base temperatures are shown in Table I.

Table I. Differences between LANL/NOAA data and analytic approximation: 55 U.S. cities/4 base temperatures.

<u>Base Temperature (°F)</u>	<u>Average Difference (AHDD)</u>	<u>Standard Deviation of Difference (AHDD)</u>
50	9	179
55	8	185
60	14	169
<u>65</u>	<u>28</u>	<u>138</u>
Overall	15	168

The numerical difference between algorithm data and LANL/NOAA data was less than 300 AHDD for 94.8 percent of the cases sampled. To put this in perspective, one AHDD costs roughly \$0.15 for typical residential space heating. Thus, annual heating cost estimates based on the algorithm are unlikely to differ by more than about \$50 from heating cost estimates based on LANL/NOAA data.

We also checked the analytic approximation for some extreme cases. A good fit is assured in these extreme cases, since the shape of the yearly temperature curve is irrelevant. This comparison is shown in Table II.

Table II. Extreme cases: Annual heating degree-days
Base Temperature 65 °F.

<u>Location</u>	<u>NOAA Data</u>	<u>Algorithm</u>
Barrow AK	20265	20344
Barter Isl. AK	19994	20052
Kotzebue AK	16039	16108
Fairbanks AK	14344	14354
Big Delta AK	13698	13696
Bethel AK	13203	13258
Talkeetna AK	11708	11761
Anchorage AK	10911	10958
Homer AK	10364	10409
Key West FL	64	0
Honolulu HI	0	0

For the data points sampled, the correlation coefficient between LANL/NOAA data and the algorithm is 0.998.

Annual Cooling Degree-Days

The algorithm can be algebraically manipulated to estimate annual cooling degree-days (ACDD) to an arbitrary base temperature. Table III compares algorithm estimates of ACDD with NOAA data for 11 U.S. locations with ACDD ranging from 0 - 6000. As with NOAA data, these estimates do not include effects of humidity.

Table III. Annual cooling degree-days (base temperature 65 °F).

<u>Location</u>	<u>NOAA Data</u>	<u>Algorithm</u>
Kwajalein Island	6121	6172
Truk Island	5955	5880
Key West FL	4756	4821
Miami FL	4095	3908
Las Vegas NV	3029	3015
Lake Charles LA	2444	2251
Albuquerque NM	1315	1254
Boston MA	699	643
Madison WI	467	320
Caribou ME	147	16
Anchorage AK	0	0

Figure 2 summarizes algorithm estimates plotted against NOAA data. Cooling degree-days are shown as negative heating degree-days simply for visual convenience.

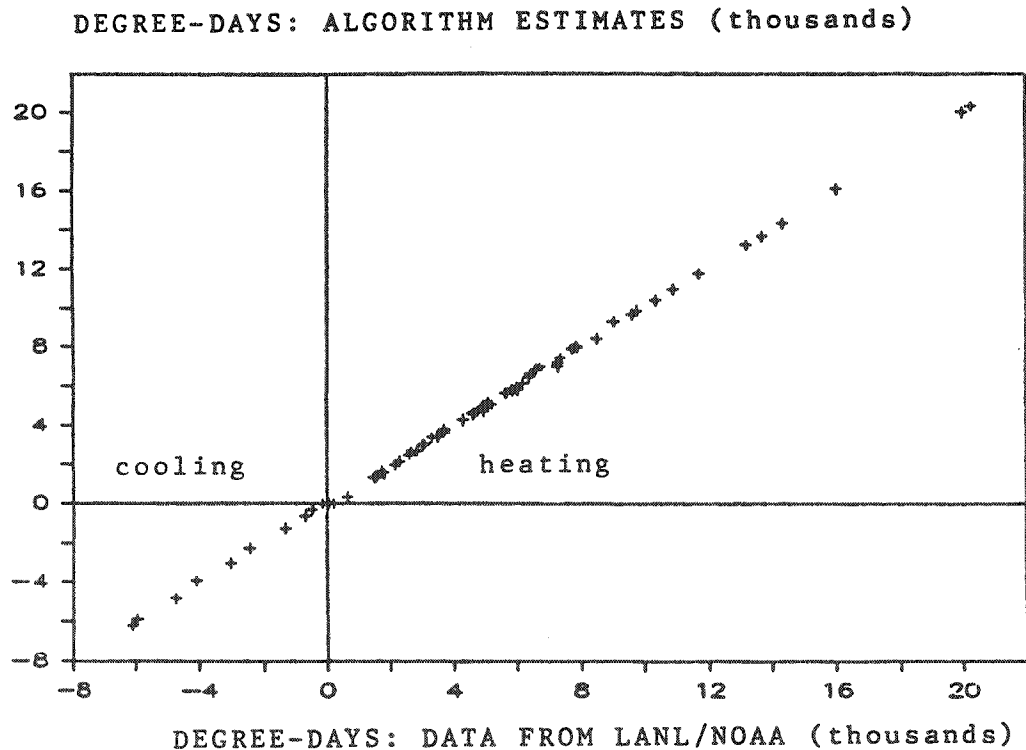


Figure 2. Comparison of algorithm and LANL/NOAA data.

CONCLUSION

Degree-days measured from a base temperature other than 65 °F are useful for estimating heating loads of energy-efficient buildings. The algorithm of this paper agrees with LANL/NOAA data with a standard deviation that is about half that of year-to-year fluctuations in AHDD, and thus may be useful for some design and estimation purposes.

SOURCES

1. Dick Holt, "How Weather Affects Seasonal Heating Cost Estimates," U.S. Department of Energy, March, 1985.
2. Los Alamos National Laboratory, Passive Solar Design Handbook; Volume III, Passive Design Analysis, July, 1982.
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4. H. C. S. Thom, "Normal Degree Days Above Any Base by the Universal Truncation Coefficient," Monthly Weather Review, Vol. 94, No. 7, 1966.