MEASURING SOLAR GAINS IN OFFICE BUILDINGS

L. K. Norford, A. Rabl, L. E. Ryan and R. H. Socolow
Center for Energy and Environmental Studies
Princeton University

ABSTRACT

Solar heat gains are an important term in the energy balance of most buildings. Knowing the magnitude of the solar gains in existing buildings is desirable in order to optimize such retrofit measures as window treatments and to verify the savings achieved by such measures. This paper discusses methods of measuring solar gains in real buildings. Since the window geometry is usually so complicated as to preclude direct measurement of transmitted radiation by means of pyranometers, one is limited to determining the solar gains from the overall energy balance of the building. This in turn presupposes accurate knowledge of the heat loss coefficient of the building. This paper examines the problem of characterizing a building by a few equivalent thermal parameters and then determining these parameters from data. As a starting point, the calculated daily and monthly solar radiation incident on the principal facades of a building is correlated with horizontal insolation. These correlations suggest that it may be difficult or impossible to isolate solar gains when one has only monthly energy consumption data, because on a monthly basis the solar gains are masked by uncertainties in the baseload and heat loss parameters of the building. But with daily or hourly data for energy consumption, temperature and insolation the solar gains can be identified. With the increasing use of energy management systems such data are becoming readily available. This paper reports the results of an effort to measure the solar gains of one office building in the Princeton area.
MEASURING SOLAR GAINS OF OFFICE BUILDINGS

L. K. Norford, A. Rabl, L. E. Ryan and R. H. Socolow
Center for Energy and Environmental Studies
Princeton University

1. INTRODUCTION

Solar heat gains are an important term in the energy balance of most buildings. Knowing the magnitude of the solar gains in existing buildings is desirable in order to optimize such retrofit measures as window treatments and to verify the savings achieved by such measures. The solar gains are also needed for a comparison between calculated and measured behavior of a building. Unlike test cells and very simple solar houses with a single aperture, the window geometry of most buildings is so complicated as to frustrate any attempt to directly measure the total solar radiation entering a building. In most cases a prohibitively large number of pyranometers would have to be placed inside the building in order to account for all the different window types and orientations and to capture the effects of varying incidence angles and of varying shadows that are cast by mullions, overhangs and other obstructions. As an alternative method we propose to determine the solar gains in real buildings from the overall energy balance.

We approach the problem from the point of view of the data analyst: given data for energy consumption and temperatures, what can one learn about solar loads on the building? This approach is distinctly different from the methodology that is appropriate at the design stage of a building. At the design stage the physical dimensions and the construction materials are known and can be input into one of the standard computer programs for calculating energy consumption [e.g. BLAST, DOE2.1, and CALPAS]. But when one wants to analyze consumption data for existing buildings, one finds that these computer programs are unnecessarily cumbersome because they require a large amount of input to describe a building, and this information may no longer be readily available. Furthermore, there is a fundamental problem with fine tuning the input to obtain agreement with the data: when the number of tunable parameters is large and many of them do not have a distinct effect on energy consumption, then there is no clear guideline as to which of the input parameters to change and how.

Therefore we adopt the method of equivalent thermal parameters, pioneered by Sonderegger [1977]. One describes the building by a simple thermal network, consisting of only a few resistances, capacitances and energy inputs, and one determines their values from a least squares fit to energy consumption and temperature data. To understand why the number of nodes of the network can be quite small, consider the following argument. If the driving forces contain only a single frequency, then the response of any linear system can be characterized by just two parameters: an amplitude and a phase [Carslaw and Jaeger 1959]. When
dealing with a single frequency any thermal circuit can thus be replaced by an equivalent circuit with just two parameters, a resistance and a heat capacity. As Sonderegger found, the response of typical buildings seems to be dominated by the diurnal frequency components of temperature and insolation, in addition to the constant average. Thus it is plausible that an equivalent circuit with two or three parameters should give a very good approximation to a real building (or one such set for each zone in a multizone building). In fact, given the finite accuracy and time resolution of typical data, it is unlikely that an attempt to determine a much larger number of parameters is even meaningful.

The paper is organized as follows. First we calculate the daily and monthly solar radiation incident on the principal facades of a building and correlate it with horizontal insolation, in order to show what kinds of effects can be expected on theoretical grounds. Then we present, in Section 2, the equations for the energy balance and for the equivalent thermal parameters of the building. In Section 3 we address the problem of characterizing the solar behavior of a building in terms of simple parameters such as an equivalent solar aperture. In the final section we illustrate the method with some preliminary results for an office building in Princeton.

2. CALCULATED SOLAR GAINS

It is instructive to begin by showing what can be expected on theoretical grounds. Solar gains arise both from absorption on the skin of the building and from radiation entering through the windows. Absorption on the skin can be analysed by means of the sol-air temperature concept of ASHRAE [1981]. With typical proportions between window and opaque surfaces, the gains through windows are likely to dominate. Let us illustrate this point with a very simple comparison of steady state heat gains per unit surface area. For an opaque portion of the building skin the solar gain is

\[ q_{\text{opaque}} = a I U/h \]  

where
- \( a \) - solar absorptivity
- \( I \) - solar irradiance on surface
- \( U \) - conductance from interior to ambient
- \( h \) - conductance from surface to ambient.

For a transparent surface of solar transmissivity \( t \) the solar gain is

\[ q_{\text{trans}} = t I \]  

if one neglects the warming of the glazing due to solar absorption in the glass itself. Typical values for an opaque wall are \( a = 0.5 \), \( U = 0.5 \) W/m² K, and \( h = 25 \) W/m² K; this implies an effective equivalent transmissivity of 0.01 for the opaque wall. That is about two orders of magnitude smaller than the transmissivity of ordinary window glass. Of course, in a real building the gains through walls and windows depend
also on the orientation and magnitude of the surfaces. But, unless the windows are very small or have low solar transmittance, the gains through the windows are indeed likely to dominate.

We have calculated the daily solar irradiation incident on the principal orientations (north, east, west and south, all vertical), and plotted it versus horizontal irradiation. The insolation is derived from a model [Collares-Pereira and Rabl, 1979], rather than directly from data because the model permits a more systematic coverage of the parameter space and thus presents a clearer picture. The model cannot be trusted in all details, but on the whole the errors are probably no worse than the uncertainties of any presently available data base. The results are shown in Figures 1 to 5 for New Jersey, whose insolation is typical of a large portion of the US. Figure 1 shows the east and west facades (they are equal in this insolation model), Figures 2 and 3 show the south facade, Figure 4 the north facade. On the south facade it is relatively easy to block direct solar radiation by means of overhangs. Therefore we have plotted the solar radiation on the south facade including and excluding direct radiation, in Figures 2 and 3, respectively. Figure 5 shows the simple average of the four principal facades (including direct radiation on the south facade). The latter is of particular interest for the analysis of a large aggregate of buildings.

The variation of solar radiation is due to two causes: geometry and weather. To explain their effects it is convenient to use the so-called clearness index $K_T$. It is defined as the ratio of terrestrial over extraterrestrial solar radiation, both taken as daily totals on the "horizontal" surface. $K_T$ ranges from about 0.05 on very heavily overcast days to about 0.75 on extremely clear days. To a good approximation this range of $K_T$ values is independent of time of year. In Figures 1 to 5 all insolation values have been calculated for the middle of each month, as labeled by the number next to each curve. For each month the curve extends from cloudy days with $K_T = 0.1$ to clear days with $K_T = 0.7$, from left to right, as the horizontal insolation is proportional to $K_T$.

The curves are not simple straight lines because of the different behavior of direct and diffuse solar radiation. The gain of the north facade is almost entirely (except around summer solstice) due to sky radiation and radiation reflected from the ground (where we have assumed a ground reflectance of 0.2). The east and west facade receive, in addition, a large amount of direct radiation. The solar gain on the south facade is disproportionately large in winter and small in summer because it is dominated by direct radiation. This effect would be further accentuated by overhangs that block direct solar radiation in summer while admitting it in winter. Figure 5 for the average over all four directions (with unshaded south facade) has an interesting feature: the day-to-day variation does not change much with time of year. In winter this average ranges from about 1 to 10 MJ/m$^2$-day, in summer from about 1 to 13 MJ/m$^2$-day.

Solar gains are very much dependent on the window orientation. As a very rough rule of thumb one might say the following:
Heast/west approx. proportional to H\textsubscript{hor}

H\textsubscript{south} winter variation with KT strong and approx. proportional
summer variation with KT only half as much as in winter

H\textsubscript{north} depends more on time of year than on H\textsubscript{hor} or KT
about 3 x larger in summer than in winter.

Monthly average values of KT for New Jersey range from about 0.4 in winter to 0.5 in summer [SERI 1980]. For a rough estimate of the monthly average solar gain one can take the monthly curves in Figures 1 to 5 and read them at the point corresponding to the monthly average KT. In Figure 5 these points lie roughly on a straight line with y-intercept 1 MJ/m\textsuperscript{2}-day and slope 0.5. The exact values do not matter for the following argument, but the approximate variation with time of year does: if monthly solar gains vary like ambient temperature, then they cannot be distinguished from temperature effects when one analyzes energy consumption of buildings. Also, the seasonal variation of the monthly averages is relatively small: their range, from approximately 3.5 to 7 MJ/m\textsuperscript{2}-day, is not large compared to their annual average of about 5 MJ/m\textsuperscript{2}-day. These two factors make solar effects difficult to determine from monthly data.

To measure solar gains of buildings one needs data for time periods when insolation and ambient temperature are not correlated. By contrast to monthly data, daily and hourly data contain episodes when solar and ambient are not correlated. Furthermore, the daily variation of insolation due to weather is much larger than the seasonal variation of monthly average values. For these reasons, for most climates, including New Jersey, one would expect to be able to measure solar gains with daily or hourly, but not with monthly data.

3. DETERMINATION OF SOLAR GAINS

Since we have ruled out a direct measurement of solar gains with pyranometers as impractical for most real buildings, we are limited to an indirect determination from the energy balance. That is not very accurate because there are several terms in the energy balance, each with its own uncertainty. On the other hand, the solar term is important only if it is relatively large compared to the other terms. Also, some of the uncertainty can be removed by taking data during unoccupied periods (vacations for residences, holidays and weekends for commercial buildings).

As a starting point we write the basic energy balance of a building in the form

\[ Q_{\text{sol}} + Q_{\text{heat/cool}} + Q_{\text{trans}} + Q_{\text{lights, equipment}} + Q_{\text{people}} + Q_{\text{ventilation/infiltration}} = \int (UA) (T_{\text{int}} - T_{\text{amb}}) \, dT + Q_{\text{transient}} \]  

(3)
where the left hand side lists all the energy inputs. The first term on the right is the steady state conductive heat flow and the second term on the right represents storage of heat in the thermal mass. The energy inputs due to lights, office equipment, fans, and heating/cooling equipment can usually be measured quite accurately, and so can the temperatures. More problematic are the gains due to people and the latent infiltration gains, but their effect can be minimized by looking at unoccupied periods without cooling.

To account for the transient term we turn to the equivalent thermal network. The simplest and most natural choices are shown in Figure 6a for two parameters and in Figure 6b for three parameters.

For the two-parameter circuit the instantaneous energy balance reads
\[ Q = \frac{(T_{\text{int}} - T_{\text{amb}})}{R} + C T_{\text{int}} \] (4)

where
- \( Q \) - total heat gain rate
- \( C \) - heat capacity (mass times specific heat)
- \( R \) - resistance between interior and ambient, equal to \( 1/(UA) \).

For the numerical analysis one replaces the differential equation by a finite difference equation
\[ \Delta Q_t = \frac{(T_{\text{int},t} - T_{\text{amb},t}) \Delta t}{R} + C(T_{\text{int},t+1} - T_{\text{int},t}) \] (5)

where
- \( \Delta t \) - time interval, labeled by subscripts \( t \) and \( t+1 \)
- \( \Delta Q_t \) - heat input during the interval, \( Q_{t+1} - Q_t \).

\( Q_t \) is the sum of solar and other gains
\[ Q_t = Q_{t,\text{sol}} + Q_{t,\text{other}} \] (6)

The exact time sequence in which the solar gain manifests itself in a building can be quite complicated. The time-of-day variation is different for different facades. Furthermore, the solar radiation is absorbed by the mass of the building, and it is transferred to the air only after some time. Equations 4 and 5 neglect this time lag by crediting the total instantaneous gain directly to the interior air.

For the three-parameter model of Figure 6b the governing differential equations are
\[ Q = \frac{(T_{\text{int}} - T_{\text{amb}})}{R} + \frac{(T_{\text{int}} - T_C)}{R_C} \] (7)

and
\[ \frac{(T_{\text{int}} - T_C)}{R_C} = C T_C \] (8)

The additional parameter \( R_C \) accounts for the resistance between interior air and thermal mass. Eliminating \( T_C \) and using finite differences we
replace these equations by the single equation

$$ΔQ_t = (T_{int,t} - T_{amb,t}) \Delta t/R + C (T_{int,t+1} - T_{int,t})$$

$$+ [ (T_{int,t+1} - T_{int,t}) - (T_{amb,t+1} - T_{amb,t})] C \frac{R_C}{R}$$

$$- C \frac{R_C}{R} [Q_{t+1} - Q_t] / \Delta t.$$ (9)

4. CORRELATION WITH SOLAR RADIATION

In order to arrive at a simple characterization of the solar behavior of a building one needs a correlation between the solar gain and the incident solar radiation. The solar radiation, in turn, could be specified in many different ways, but the hemispherical solar radiation on the horizontal surface is by far the most convenient because of the general availability of such data.

If solar gains were proportional to hemispherical horizontal insolation $H_{hor}$, one could define an equivalent effective solar aperture of a building by the simple equation

$$Q_{sol} = A_{hor} H_{hor}.$$ (10)

Unfortunately this may not generally be the case. Figures 1 to 5 show the solar gain through vertical windows is not simply proportional to horizontal solar radiation (or to any other single measure of solar radiation such as direct or diffuse radiation). Because of complicated geometric effects and because of the varying relation between direct and diffuse radiation, the behavior of solar gains is different for different facades and for different times of year.

On the basis of first principles one would expect that the instantaneous solar gain at any moment could be related to the direct and diffuse solar irradiances $I_{dir}$ and $I_{diff}$ in terms of two effective apertures $A_{dir}$ and $A_{diff}$ as

$$Q_{sol} = A_{dir} I_{dir} + A_{diff} I_{diff}.$$ (11)

$A_{diff}$ would be a simple number to the extent that the diffuse radiation is isotropic. But $A_{dir}$ can be a strongly varying function of solar altitude and azimuth. While $A_{dir}$ could in principle be calculated, its measurement may be difficult.

One could try an analogous equation to relate daily solar gains to the daily direct and diffuse solar irradiation values $H_{dir}$ and $H_{diff}$

$$Q_{sol} = A_{dir} H_{dir} + A_{diff} H_{diff}.$$ (12)

But one obvious problem with this approach is the difference between days with clear mornings and cloudy afternoons and days with cloudy mornings and clear afternoons. Thus Equation 13 can at best be an approximation
for the average over many days.

It is difficult to tell in advance how many parameters one needs to characterize the solar aperture of a building. One will probably have to make a compromise between accuracy and simplicity. The best approach seems to be to start with a single solar aperture $A_{hor}$ and add more coefficients if necessary.

5. AN EXAMPLE

We illustrate the method with data from an all-electric office building in Princeton with 12,000 m$^2$ floor space [Norford et al. 1984]. The building is being instrumented with about 50 data channels, and some preliminary data are available for April, 1984, a time when the building was not yet occupied. There is considerable uncertainty in the data because at that time the main electricity meter could only be read manually and the resolution was about one third of the total consumption during each 12 hour period (the meter records daytime and nighttime consumption separately, with cutoffs at 7 am and at 7 pm). Nonetheless we are able to unambiguously identify the solar gains.

First we used nighttime temperature and total electric consumption data to determine the heat loss coefficient and the effective heat capacity of the building as $(UA) = 11.5$ kW/C and $(mc) = 633$ kWh/C, based on the two parameter thermal network of Figure 6a. Then we inserted the daytime (7 am - 7 pm) temperature values into Equation 5 to obtain $Q_t$ for each of the days listed in Table I (note that the time interval in this case is $t = 12$ hr). Since the buildings were unoccupied at the time, the solar gain is simply the difference between the total $Q_t$ and the electricity consumption. All the terms are listed in Table I. The solar gains are plotted versus hemispherical horizontal irradiation for the same time period in Figure 7. Despite the large uncertainty in the data, the plot displays a clear trend. In fact, it seems as if the simple model of Equation 10 fits this building quite well, with an effective equivalent solar aperture of $A_{hor} = 300$ m$^2$. These values for the solar gain are consistent with calculations made by Princeton Energy Group [PEG 1983].

For the work performed to date, the two-parameter thermal network model has been used in lieu of the three-parameter model. The latter, which includes a thermal resistance separating room temperature from mass temperature, is more accurate when a building's carpeting and drop ceiling are installed, thereby increasing the value of this resistance, and when the difference between room and mass temperature is important, as with hourly simulations. At this time, the Enerplex floor slabs are bare and our work has consisted of daily analyses. We plan to apply our models to other buildings, using the two or three-parameter models as appropriate.
ACKNOWLEDGEMENTS

We would like to thank The Prudential Insurance Company of America and the United States Department of Energy for financial support.

REFERENCES


Figure 1. Daily solar irradiation on east or west-facing vertical wall, as a function of month (labeled by number) and of horizontal irradiation $H_{\text{hor}}$.

Figure 2. Daily solar irradiation on south-facing vertical wall, as a function of month (labeled by number) and of horizontal irradiation $H_{\text{hor}}$. Direct radiation is included.
Figure 3. Daily solar irradiation on south-facing vertical wall, as a function of month (labeled by number) and of horizontal irradiation $H_{hor}$. Direct radiation is excluded.

Figure 4. Daily solar irradiation on north-facing vertical wall, as a function of month (labeled by number) and of horizontal irradiation $H_{hor}$. 
Figure 5. Average of daily solar irradiation on east, south, west and north-facing vertical walls, as a function of month (labeled by number) and of horizontal irradiation $H_{\text{hor}}$. Direct radiation is included for the south wall.

(a) One resistance, one capacitance.

(b) Two resistances, one capacitance.

Figure 6. Equivalent thermal circuits for a building.
Average Daytime (7 AM to 7 PM) Energy Balance for ENERPLEX South
Assuming UA = 11.5 kW/C and mc = 633 kWh/C

<table>
<thead>
<tr>
<th>date</th>
<th>Ti - To</th>
<th>dTi/dt</th>
<th>UA(Ti - To)</th>
<th>modTi/dt</th>
<th>Qelec</th>
<th>Qsolar</th>
<th>Hhor (Wh/m²)</th>
</tr>
</thead>
<tbody>
<tr>
<td>4/14/84</td>
<td>9.77</td>
<td>-.087</td>
<td>112.355</td>
<td>-55.03</td>
<td>55</td>
<td>2.33</td>
<td>353</td>
</tr>
<tr>
<td>4/15/84</td>
<td>6.48</td>
<td>.00363</td>
<td>74.52</td>
<td>2.30</td>
<td>63</td>
<td>13.82</td>
<td>416</td>
</tr>
<tr>
<td>4/16/84</td>
<td>4.08</td>
<td>.0759</td>
<td>46.92</td>
<td>48.01</td>
<td>33</td>
<td>61.93</td>
<td>671</td>
</tr>
<tr>
<td>4/20/84</td>
<td>1.98</td>
<td>.2245</td>
<td>22.77</td>
<td>142.00</td>
<td>56</td>
<td>108.77</td>
<td>4020</td>
</tr>
<tr>
<td>4/21/84</td>
<td>2.62</td>
<td>.2688</td>
<td>30.13</td>
<td>170.02</td>
<td>56</td>
<td>144.15</td>
<td>6135</td>
</tr>
<tr>
<td>4/22/84</td>
<td>8.02</td>
<td>.1342</td>
<td>92.23</td>
<td>84.88</td>
<td>56</td>
<td>121.11</td>
<td>4150</td>
</tr>
<tr>
<td>4/25/84</td>
<td>2.33</td>
<td>.1135</td>
<td>26.795</td>
<td>71.79</td>
<td>60</td>
<td>38.58</td>
<td>1845</td>
</tr>
<tr>
<td>4/26/84</td>
<td>-3.91</td>
<td>.3828</td>
<td>-44.965</td>
<td>242.12</td>
<td>60</td>
<td>137.16</td>
<td>6084</td>
</tr>
<tr>
<td>4/27/84</td>
<td>1.86</td>
<td>.2432</td>
<td>21.39</td>
<td>153.82</td>
<td>60</td>
<td>115.21</td>
<td>5568</td>
</tr>
</tbody>
</table>

Table I. Data for Enerplex South office building.