Development and Testing of Model Predictive Control for a Campus Chilled Water Plant with Thermal Storage

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ABSTRACT

A Model Predictive Control (MPC) implementation was developed for a university campus chilled water plant. The plant includes three water-cooled chillers and a two million gallon chilled water storage tank. The tank is charged during the night to minimize on-peak electricity consumption and take advantage of the lower ambient wet bulb temperature. A detailed model of the chilled water plant and simplified models of the campus buildings were developed using the equation-based modeling language Modelica. Steady state models of the chillers, cooling towers and pumps were developed, based on manufacturers' performance data, and calibrated using measured data collected and archived by the control system. A dynamic model of the chilled water storage tank was also developed and calibrated. A semi-empirical model was developed to predict the temperature and flow rate of the chilled water returning to the plant from the buildings. These models were then combined and simplified for use in a MPC algorithm that determines the optimal chiller start and stop times and set-points for the condenser water temperature and the chilled water supply temperature. The paper describes the development and testing of the MPC implementation and discusses lessons learned and next steps in further research.

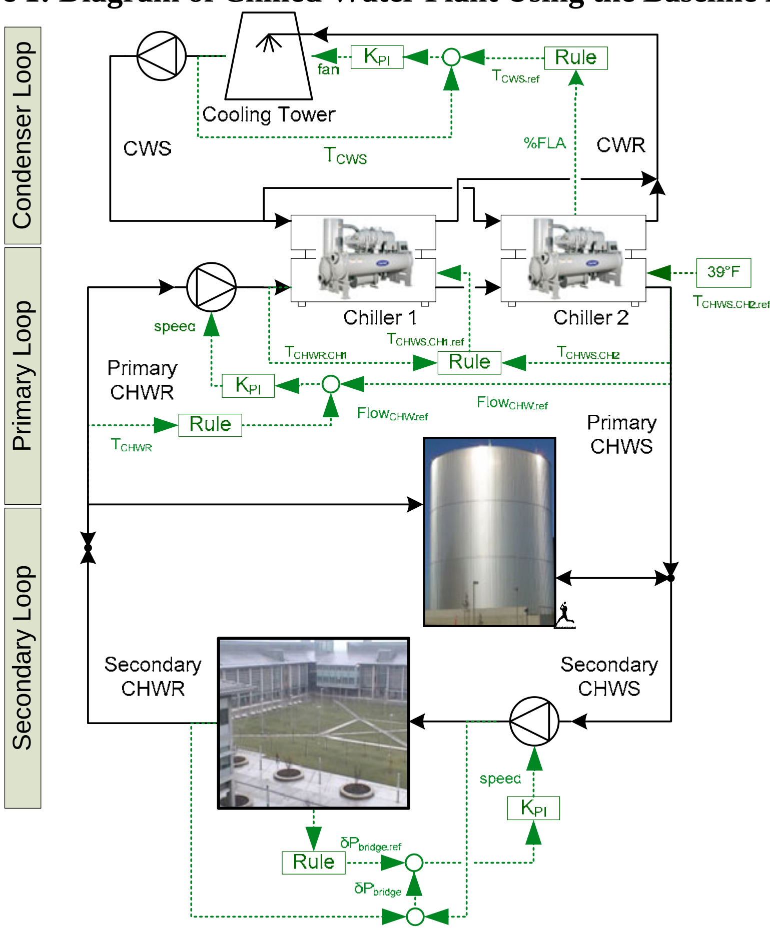
Introduction

Technologies for energy efficiency improvement in buildings are central to the development of marketable design approaches for net zero energy commercial buildings by 2025, which is a strategic goal of the Department of Energy (DOE) Buildings Technologies Program. Heating, ventilation, and air conditioning (HVAC) account for 27% of the energy consumption and 45% of peak electrical demand in commercial buildings. Approximately 5% of the floor area of US commercial buildings is cooled by central chillers or district chilled water plants (CBECS 2003). Thermal energy storage (TES) can be used to reduce eletricity costs and (in some cases) energy consumption for these chilled water systems by generating chilled water overnight when the weather and electriciy rates are advantageous. Simple scheduling control of TES is able to capture significant cost savings (depending on the utility rate structure) and often some energy savings (depending on the diurnal ambient swings and the losses in the TES tank). This paper presents the development and testing of a Model Predictive Control (MPC) implementation for a campus chilled water system with TES, in an effort to save more energy than possible with simple scheduling control by taking advantage of weather forecasts to optimize the charging window and system loop temperature set-points for each night.

UC Merced Chilled Water System Description

The UC Merced campus is used as a case study. It has a central chilled water system with a two million gallon chilled stratified TES tank. The main components of the UC Merced chilled water system are shown in Figure 1: a condenser loop, a primary loop, a secondary (campus) loop, and multiple tertiary (building) loops. Water cooling is performed by the chillers and cooling towers in the primary and condenser loops. The chilled water is stored in a stratified TES tank and distributed to the buildings throughout campus via the secondary loop. The tertiary loops use pumps and valves within each building to distribute the chilled water for consumption by the fan coils and air handling units. The chilled water is warmed by the air-side cooling load of the buildings and returned to the secondary loop. During tank charging, cool water from the chillers enters the bottom of the tank and warm water from the top of the tank is returned to the chillers. During tank discharging, cool water from the bottom of the tank is supplied to the campus, and warm from the campus returns to the top of the tank. The energy management and control system (EMCS) provides the supervisory control that coordinates two electric chillers (currently only two of the three are used at any given time), cooling towers, and multiple pumps in the overnight charging of the tank.

Figure 1: Diagram of Chilled Water Plant Using the Baseline Policy



Controls Challenge

There are four main control decisions affecting energy use for each overnight charging period: the charging start time, the charging length, the condenser water supply temperature (CWS) set-point, and the chilled water supply temperature (CHWS) set-point. The existing control policy is to completely charge the tank on nights when the charge level is below 50% of the full capacity (and otherwise not to charge at all), to start charging at about 10pm, and to use a constant CHWS set-point of 39°F and a CWS set-point of 57-60°F (varying linearly with load).

UC Merced is currently enrolled in a utility plan where the electricity price depends on the period of day. During the summer (May 1 – Oct 31), the prices are 13.6 c/kWh during the period 12:00-18:00, 9.2 c/kWh during the periods 8:30-12:00 and 18:00-21:30, and 7.4 c/kWh during the overnight period 21:30-8:30. As long as the charging starts after 21:30 and ends before 8:30, the electricity rate is always the same: this benefit of the TES system is already captured by the existing control policy. But energy savings compared to the existing policy may be possible by charging the tank to less than full capacity, charging during the lowest wet bulb temperature window, charging more on cooler nights and less on hotter nights, and adjusting the CWS and CHWS set-points to maximize the system COP at different wet bulb temperature and return temperature conditions.

A new control strategy is desired that can take advantage of these potentials for energy savings. The current state of control technology within building systems would not use weather or cooling load forecasting to dynamically manage chilled water storage. Instead, static, heuristic policies are typically employed, defining rules for the diverse conditions and modes of operation encountered during hourly, daily, weekly, and seasonal operation. The performance of the resulting control system is highly dependent on the expertise of the control designer or operator. Instead of attempting to devise an expanded set of rules for this system, MPC was used.

Model Predictive Control (MPC)

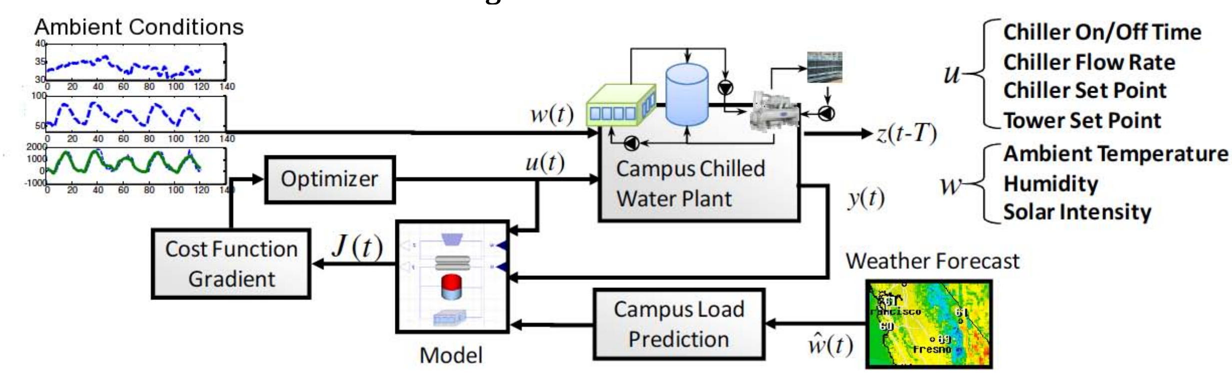
The essential idea of MPC is to use a system model and an optimization algorithm to determine the control set-points. At each control time-step, starting at the current state and using weather predictions, an open-loop optimal control problem is solved over a finite horizon. The solution provides a sequence of optimal inputs over the horizon, only the first element of which is applied to the system during the following sampling interval. At the next time step a new optimal control problem based on new measurements of the system state is solved over a shifted horizon. For complex constrained multivariable control problems, MPC has become the accepted standard in the process industries (Morari & Lee 1999; Mayne et al. 2000; Borrelli 2003); its success is largely due to its unique ability to systematically, simply and effectively handle constraints on control and states. The idea of MPC for supervisory control of buildings was noted at least as early as 1988 (Kelly, 1988), but did not receive much research attention until the past decade. A modest number of case studies have been performed for various systems. Noteworthy precedents to this study of MPC for energy minimization through thermal energy storage include the work of Henze et al (2004, 2005a, 2005b), Clarke et al (2002) and Kummert et al (2005). A more complete history of MPC research for buildings is available in Coffey et al (2010).

A MPC scheme was developed and tested for the UC Merced campus chilled water system. Detailed component models were developed in the Modelica language using elements from the Buildings library (Wetter 2009a; Wetter 2009b). These were used as the basis of simplified Matlab models and lookup tables for use within the controller. The MPC determined the optimal CWS and CHWS set-points and the charging start and stop times to minimize energy consumption over a 3-day prediction horizon. The models, prediction and optimization configuration are described below, followed by a description of an experimental test and results. The main contributions of this work to the field of MPC research for buildings are as follows: the addition of another experimental implementation to the small collection experimental case studies in the literature; the development of new controls-appropriate models for campus chilled water systems; and the treatment of the terminal constraint.

MPC Development

The scheme of the MPC implementation is depicted in Figure 2. This section describes the components of this MPC scheme, with particular focus on the development of the system model.

Figure 2 MPC Scheme



System Model

A detailed system model was developed in Modelica and used as the basis for the faster-running models and lookup tables in the online system model. Detailed mathematical descriptions of the individual components are presented in Haves et al. (2010), with just some of the most pertinent aspects of the model noted here.

The storage of chilled water in the TES tank can easily meet the current campus cooling loads for more than a day. Thus, any decision made with respect to quantity and temperature of the chilled water stored in the storage tank affects the performance of the entire system over a relatively long time horizon. In contrast, the chilled water plant components—such as the chillers, cooling towers, and pumps—have very short time constants. In fact, the 15 minute sampling interval of the energy management control system generally results in undersampling of the transients in these components. As a result, the model focuses on the dominant dynamics of the TES tank and the campus cooling load generated by the buildings, and the operation of the much shorter time constant system components is treated as quasi-static. As such, the only state variables in the system model are in the TES tank.

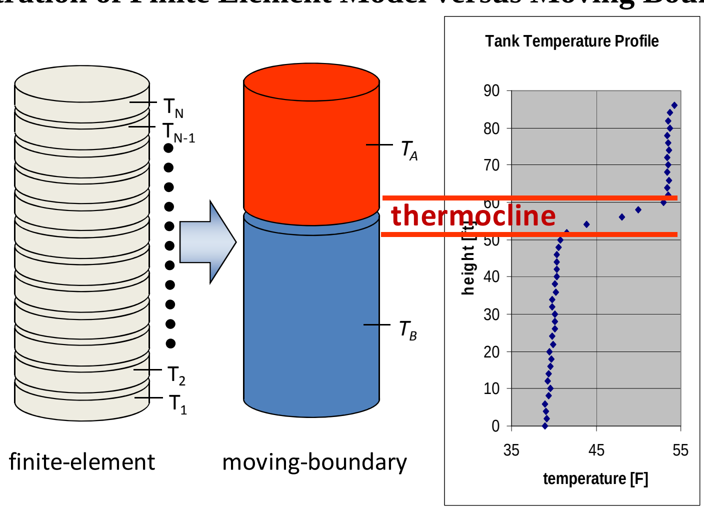
Lower-level controllers modulate the operation of the chillers and cooling towers in order to achieve the desired condenser water supply temperature from cooling towers, the desired mass flow rate of chilled water through the chillers and the desired chilled water supply temperature.

The system model assumes that there is no tracking error between the controlled variables and their set-points. In reality, there is significant tracking error in the plant, as discussed later.

In the detailed model, the chillers and cooling towers are modeled using standard methods for building energy simulations, and calibrated using manufacturer specifications and measured data from the campus. The temperature profile in the tank is modeled by discretizing the tank into a number of layers. A new model was developed to estimate the return temperature and flow rate from the campus, given the campus cooling load, chilled water supply temperature and ambient temperature. A purely empirical model was insufficient because the historical operation kept a constant supply temperature close to 39^{o}F, but this was to be varied by the MPC. The model was based on an effectiveness-NTU model, with the effectiveness varying as a function of the water and air flow rates and the load. The model was calibrated using historical data and the data from a 3 day experiment in September 2009 when the supply temperature was increased by 2^{o}F.

Based on this detailed model, a faster-running model was created for use in the MPC. Lookup tables were created using the detailed models of the chillers, towers and campus heat exchange. A new simplified model was created for the TES tank, with the goals of decreasing the computing time and the number of state variables, while still accurately capturing three essential aspects: (1) the total stored cooling capacity, (2) the temperature of the water supplied to the campus, (3) the temperature of the water returned to the chiller. The three aspects of interest are effectively embedded in the temperature profile of the stratified tank. In the simplified model, the thermocline between the warm and cool masses of water is treated as a natural moving boundary (the steep measured gradiant between hot and cold (Figure 3) is treated as a step change). The cool and warm water are treated as lumped masses, thus requiring only three dynamic states for the tank model, i.e. one for the position of the thermocline and two for lumped mass temperatures. In Haves et al. (2010), the reduced-ordered model is shown to correlate well with campus measurements. The calibration and validation process also revealed that the amount of stored cooling capacity lost due to thermal losses is minimal.

Figure 3: Illustration of Finite Element Model versus Moving Boundary Model



Disturbance Predictions (Weather Forecasts and Campus Load Prediction)

In order to obtain weather forecasts for use by the MPC algorithm, the National Digital Forecast Database (Glahn & Ruth 2003) was used. A Simple Object Access Protocol request was used to query the NDFD and parse and pass the weather forecast for UC Merced to the MPC algorithm. For the experiments described below, only the forecasted dry bulb temperature and relative humidity were used. In the future, additional forecasted variables, such as cloud cover, could also be used.

A low-order calibrated model was developed to predict the campus cooling load given the predicted ambient temperatures and the time of day, day of week and day of year. (Note that since this model is not part of the optimization loop, it did not have to go through the same lookup table or simplification process.) The model is described in detail in Haves et al. (2010). Solar loads are calculated based on the latitude, time of day, day of year and calibrated parameters that relate insolation to solar loads for the campus. Internal heat loads are estimated based on calibrated schedule parameters. The solar and internal loads and the ambient temperature are then used in a single-zone lumped-parameter thermal model to calculate the cooling load. The resulting model has 18 parameters that are calibrated with meaured campus load data. In Haves et al. (2010), the model is shown to correlate well with measured data.

Optimization and Constraints

The MPC open-loop finite time optimization problem at each controller time step is shown in Equation 1

$$J^{å} = \min_{u_{t\to t+N|t}} P(x_{t+N|t}) + \sum_{k=0}^{N-1} L(x_{t+k|t}, u_{k+t|t}, d_{t+k|t}) \quad (1a)$$

$$\text{subject to} \quad x_{t+k+1|t} = g(x_{t+k|t}, u_{k+t|t}, d_{t+k|t}), \forall k = 0,1,..., N-1 \quad (1b)$$

$$x_{t+k|t} \in \Xi, \forall k = 1,2,..., N \quad (1c)$$

$$u_{t+k|t} \in \Upsilon, \forall k = 0,1,..., N-1 \quad (1d)$$

where u are the controlled inputs (CWS set-point, CHWS set-point and the chilled water supply mass flow rate set-point), x represent the system states (height of the thermocline and temperatures of the cool and warm water in the tank), and d represent the external disturbances (ambient temperature and campus load). $P(x)$ is the terminal cost (not used in this case) and $L(x,u,d,t)$ is the stage cost (modeled energy consumption), which captures the performance objective to be minimized. The function $g(x,u,d,t)$ is a compact representation of the state update dynamic equation, which describes the cooling plant and buildings (i.e. the system model). The control inputs u and the states x are subject to the operational constraints (1c) and (1d) which avoid system malfunctioning (such as chillers surging) and ensure that the campus cooling demand is satisfied over the prediction horizon. All variables use the following time indexing: $x_{t+k|t}$ denotes the state vector at time $\mathrm{t}+k$ predicted at time t obtained by starting from the current state $x(t)$ and applying the input sequence $u_{t\to t+N|t}$ and disturbance prediction $d_{t\to t+N|t}$ (weather and campus load) to the system model.

A 72 hour prediction horizon was chosen for several reasons. First, the weather forecast data is available in 3 hour increments out to 72 hours and in 6 hour increments out to 168 hours (Glahn & Ruth 2003). Second, longer prediction horizons are met with greater challenges in the realization of a real-time MPC algorithm. Third, a 72 hour prediction horizon was deemed sufficiently long to take advantage of fluctuations in weather and occupancy (e.g. weekends) via thermal energy storage.

A reasonable selection of prediction horizon itself is not enough to enable the real-time implementation of MPC. A move blocking strategy (Kerrigan, 2007) is further introduced to reduce the computational complexity of problem (1) while retaining the persistent feasibility of the resulting MPC. The main idea is to reduce the number of optimization variables by fixing the control inputs to be constant over several time steps. For technical details refer to Kerrigan (2007) and references therein. In this case study, in order to optimize the charging schedule, the block length varies according to the charging start time and end time, and during each charging period the control inputs remain constant.

The handling of the terminal constraint $X_{t+N|t}$ in this case is of particular interest. Without either a terminal constraint or terminal cost, the control trajectory that minimizes system energy would usually result in a completely uncharged tank at the end of the control horizon. In order to ensure that the tank will always have enough charge to meet demand on the day following the last day in the prediction horizon, a robust invariant set (the set of all system states for which any expected future disturbance can be handled by the controller without a violation of state constraints) was calculated by using the detailed TES tank model. This was used as the terminal constraint.

The optimization problem (1) is solved by using a sequential quadratic programming method. The commercial software NPSOL® (Gill, online) has been used for such purpose. The solution requires about 20 minutes of computing time on a standard laptop. Each evening, the computed first time-step set-points for CHWS, CWS, charging start time and charging length were given to the operators to use for that overnight charging period.

MPC Testing

Comparisons against baseline are made in terms of the coefficient of performance (COP), which is calculated as the ratio of cooling generated to electrical energy consumed. The COP was chosen as metric that is more or less independent of the scale of the chilled water plant, and provides a means of comparing performance over days or weeks that have different loads. The historical data for the average overnight COP shows a significant amount of fluctuation in response to different overnight ambient wet bulb temperatures, campus return temperatures and the maximum primary loop flow rate allowed through the chillers (which has a large effect on the COP and which changes over time as problems occur such as broken pumps and stuck dampers). So a regression model was developed for the COP as a function of these three variables, providing a more accurate and appropriate baseline than simply using an average over the previous weeks. The details of this regression are provided in Haves et al. (2010).

Two MPC implementation experiments were carried out at the UC Merced campus. The first was a week long test of the MPC in June 2009. A miscalculation in the MPC algorithm implementation allowed a suboptimal charging window length to affect the overall COP. Subsequently, the sub-optimal operation manifested no improvements in overall performance. However, the development and testing process for this first experiment resulted in the identification and replacement of a malfunctioning flow rate sensor in a campus building, resulting in pump energy savings at that building and an increase in the temperature of the chilled water returning from the campus, increasing the cooling system COP. Additionally, the plant operators learned from the MPC set-points in this first experiment that the system COP can increased by adjusting the standard condenser water set-point range from 57-60^{o}F to 65-66^{o}F. The performance improvement associated with this CWS set-point change is difficult to quantify from the measured data, since the flow rate sensor was fixed at the about same time as the CWS set-point change, but regression analysis suggests the improvement in COP to be approximately 1.5%.

With improved models and algorithms, a second MPC experiment was carried out during the week of Oct 5-11, 2009. This was not an ideal experimental week, as the cooling load was much lower than during the summer. On the first night, for example, the tank was already nearly fully charged since there was so little load during the preceding day, so the MPC determined that no charging should occur that night (which was the same decision that the existing control policy would have made). Various other problems were encountered during the week ranging from set-point tracking problems to chiller malfunctions. Details of one charging period, the night of Oct 9, are provided in Figure 4, showing how the MPC elected to start charging later in the night than the standard control would have, taking advantage of the lower ambient wet bulb temperature. Figure 5 shows the variation in condenser water temperature set-points over different charging periods, as well as the variation in the charging period start time and length specified by the MPC. Both figures also illustrate some of the set-point tracking problems that were encountered during the experiment.

Figure 4: Details of MPC Operation, Charging Period on the Night of Oct 9, 2009

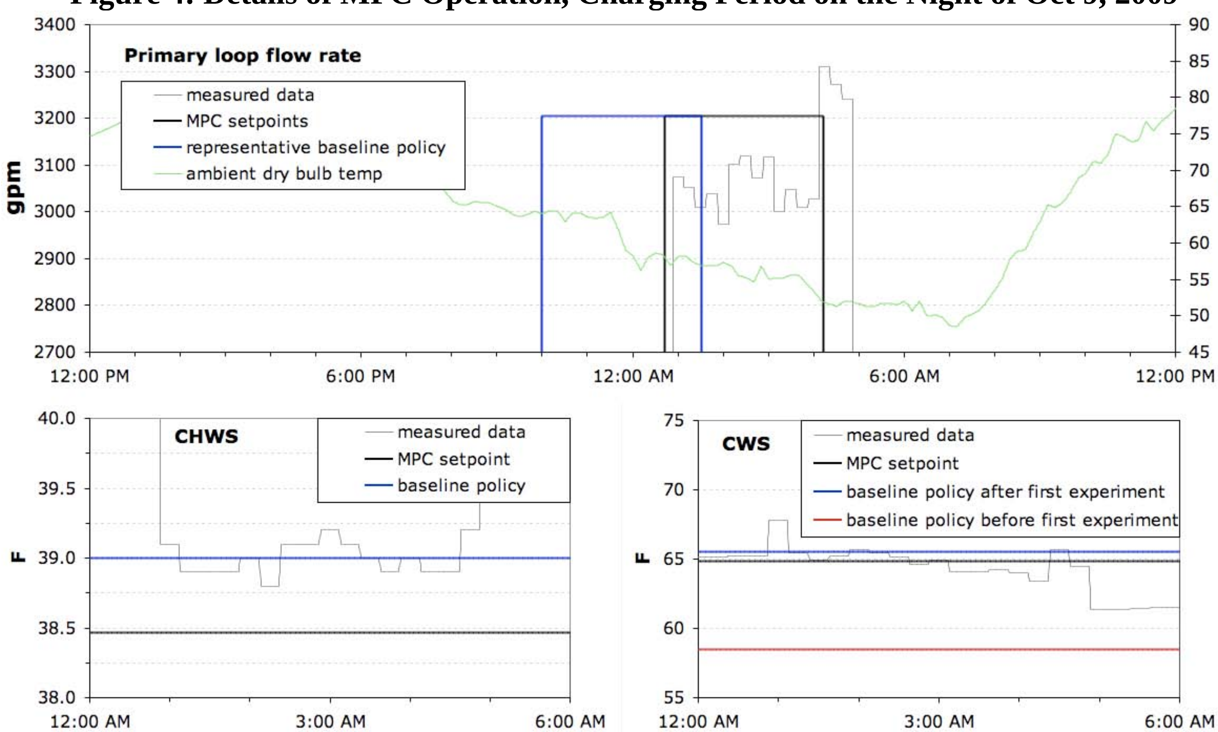
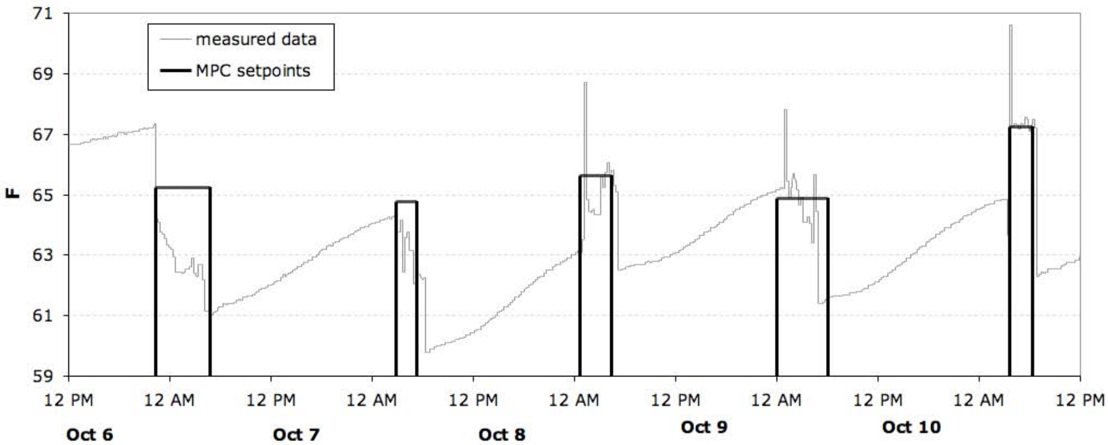
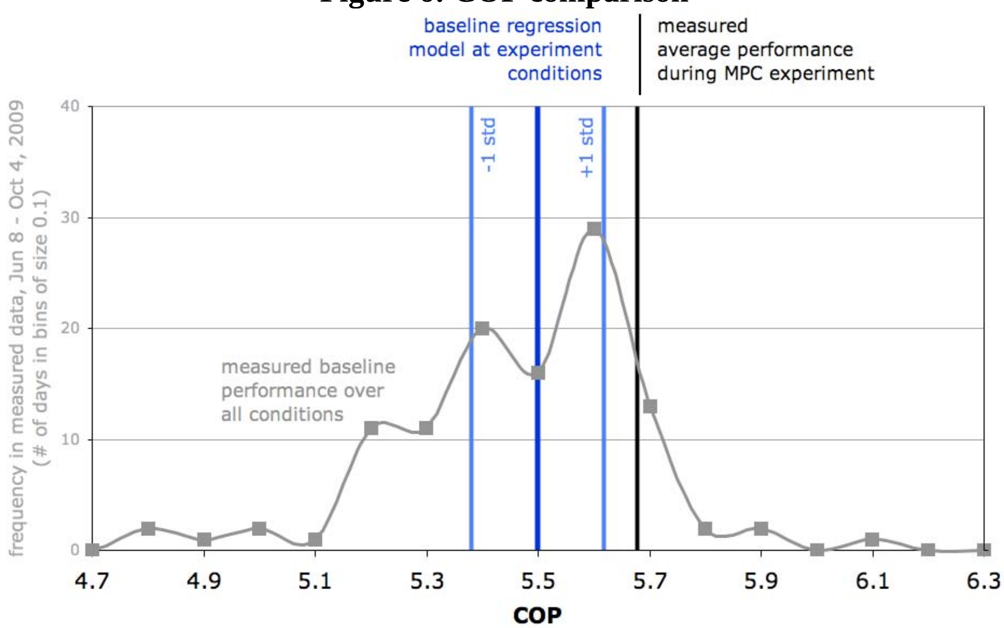


Figure 5: Condenser Water Supply Temperature Set-Point and Actual, Oct 6-10, 2009



The experimental results show a small improvement in COP over the baseline policy, as shown in Figure 6, but it is difficult to draw any strong conclusions about the energy savings potential for MPC with this system since there are just four experimental data points to consider in the comparison. (The charging period on the night of Oct 7 was not considered because a chiller malfunctioned and the operators returned to standard control set-points for the rest of the charging period.) The comparison with the baseline regression model under the experimental conditions shows an improvement in COP of 3.1% +/- 2.2%. (Note that the baseline regression in this case was based only on the data after the first experiment and before the second, so the COP comparison does not include the savings from the operators' CWS set-point change after that first experiment.)

Figure 6: COP comparison



Discussion

Although the energy savings through the MPC test itself were limited, the process of MPC development and testing brought about a number of other benefits for the system operation. Model development and data collection resulted in the identifying and remedying of various problems in the system. It was determined that the CHWS set-point and the chilled water flow rate can be used to limit the chiller loading to prevent chiller surging. The identification and analysis of inconsistencies in central and buildings-level return temperature data led to the flow rate sensor fix noted earlier. Although the types of problems uncovered may differ significantly from project to project, similar benefits of detecting and diagnosing problems are expected from the use of a comprehensive model-based approach including modeling, validation, and MPC.

In addition to these commissioning benefits, the MPC development and testing process was also able to identify simple ways to improve the heuristic control policy currently used by the operators. It was found that operating the chillers near full load was a key factor in maximizing system efficiency. In order to maximize the chiller load, one must maximize the temperature difference across chillers and the chilled water flow rate through the chillers. However, the admissible CHWS and flow rates are bounded, so the chiller loading is constrained and often determined by the temperature of the primary loop chilled water return, which is primarily a function of the temperature near the top of the TES tank. This temperature falls substantially as the thermocline approaches the top of the tank, reducing the available chiller loading. Thus, overcharging the TES tank can be detrimental to the chilled water plant efficiency and should be avoided. It is also expected that additional savings are available by concentrating low loads on a single chiller rather than spreading it across multiple chillers. And as noted earlier, the plant operators learned from the MPC set-points in this first experiment that the system COP can be increased by increasing the standard condenser water set-point range. Further research is required to determine how much energy could be saved through the addition of simple rules like these to the existing policy, based on the lessons learned in the MPC study.

The above-described MPC implementation is not robust in the sense that neither input uncertainties nor model error are taken into account. This could be addressed by designing a robust MPC scheme (Borrelli 2003; Witsenhausen 1968; Bemporad, Borrelli & Morari 2003). A typical robust MPC scheme involves solving a min-max problem to optimize robust performance (the minimum over the control input of the maximum over the disturbance) while enforcing input and state constraints for all possible admissible bounded disturbances. Also, the MPC implementation does not consider system faults. Robustness to faults can be indirectly obtained by detuning the controller (reducing the weights) at the price of a reduced performance or can be systematically and easily taken into accounts by switching to a different model $g(x,u,t,d)$ and constraints X and U when a fault is identified. Given the uncertainties and system faults apparent in this case study, and their prevalence in building systems in general, future research on robust MPC for buildings applications is recommended.

In order for the approach to be commercially viable, the time and expertise intensity required for the development and testing of the MPC implementation must be greatly reduced. Ultimately, a streamlined tool chain for the development and implementation of model-based building control and commissioning stands to address part of these challenges. As shown in this case study, MPC development and fault diagnostics can be two mutually reinforcing activities. In order to facilitate such activities, the development and dissemination of controls-oriented models for building system components, such as those developed during this project, is a first step. The complexity of building systems calls for methods that enable detailed component models to be aggregated into lower-order models for coordination at higher levels. And the configuration of advanced control and monitoring techniques should ultimately be transparent to the installers and operators in order to facilitate communication and understanding and to save energy.

Conclusions and Recommendations

A Model Predictive Control (MPC) scheme was successfully developed and implemented for the UC Merced chilled water system. New component models were developed in the process, various problems were uncovered and fixed in the system operation, and some simple control policy changes were found that can improve the system COP. Experimental implementation showed marginal system efficiency improvements over the baseline, but there are not enough data points to draw very strong conclusions from the results. The process of developing and implementing MPC for such systems needs to be made faster and simpler in order for better tests to be carried out and for MPC to be commercially viable. The models developed and lessons learned from this implementation should help in that respect.

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