

# Daylighting Commercial and Educational Rooms to 750 Lux with Stationary Projecting Reflector Arrays (SPRA): A Simulation

**Michael R. Stiles, Synertech Systems Corporation**  
**Ross McCluney, Florida Solar Energy Center**

Illuminating performance is assessed for a class of passive beam daylighting systems known as stationary projecting reflector arrays (SPRA). The reflectors are oriented to provide glare-free reflection of solar flux to the ceiling of a south-facing room typical of educational and commercial settings. A first-order model of illumination is derived for computing the contributions of SPRA to a work plane at desk level. The model is combined with standard daylighting calculations to simulate room illumination levels at three latitudes in the United States. Two different conservation strategies for modulating fluorescent lights are applied to the information from the daylighting simulations, using a set-point illumination at the work plane of 750 lux. The computed savings due to SPRA in electric lighting energy were about the same for all three latitudes. A strategy of shutting fluorescent lights off, when SPRA daylighting achieves the set point, reduces consumption of electricity for lighting by an annual average of 25%. A strategy of dimming the fluorescent lights to maintain work plane illuminance at the set point reduces electric lighting needs by an annual average of about 44% when SPRA is used. The simulations show that incorporating SPRA achieves a 20-25% greater savings than base-case conditions without SPRA. These results suggest that SPRA beam daylighting can replace significant quantities of electric lighting energy in sunny climates.

---

## Introduction

Numerous attempts have been made over the last several decades to supplant electric lighting with well-distributed daylight in rooms with vertical windows in one wall. The problem is that natural illuminance levels are quite high immediately adjacent to the window wall but drop off rapidly with distance away from this wall, falling to as low as 15% only 4 or 5 meters in (Kaufman and Christensen 1984).

Various measures have been developed to overcome this problem, with varying degrees of success. Light shelf systems are effective at reducing illumination levels at the window wall, but are not particularly effective at distributing the reflected flux deep into the room (Benton 1986). Active systems are expensive and require more maintenance than totally passive ones. Horizontal and vertical reflector systems offer very little control over the directions of reflected flux, and often do not reflect the solar beam at angles useful for room illumination (Stiles 1993).

The cylindrical version of the optical solar tracking system proposed by one of the authors is totally passive (POST),

but the quantities of flux delivered into the room are modest, and the system requires a sizeable apparatus to be located outside the window (McCluney 1983). It does, however, offer the advantage of working best on north-facing window walls, and can be considered a complementary system for the one explored in this paper.

Stationary projecting reflector arrays (SPRA) proposed for southerly-facing rooms are comprised of passive reflectors mounted between glazings in windows (Stiles 1992a; 1993; 1994). These systems offer well-defined control over the directions of the reflected solar beam throughout the year, and can be fabricated from relatively inexpensive materials common in the window manufacturing industry (Stiles 1992b). Previous investigations have been concerned with the directional properties of SPRA daylighting systems. The present paper explores their illuminating performance.

The primary goal of this paper is to introduce the capability of SPRA systems to supplant electric lighting in a type of south-facing room often found in commercial and educational facilities. Estimates of savings in electric lighting

are made possible by a mathematical model of SPRA's contribution to work plane illuminance. This model provides the information needed to simulate savings when conservation strategies are applied to modulate electric lighting levels according to the available daylight.

### A Model of Illumination from Specular Reflectors

This model is derived for illumination from flat rhomboidal or rectangular reflectors. The derivation is based on standard physical principles of the reflection of luminous flux (McCluney 1994). In Figure 1, a reflector of area  $S$  is oriented within an appropriately specified coordinate system. Direct solar illuminance from a clear sky is incident on the reflector at an angle  $B$  and is given by  $E_{dn}$ :

$$E_{dn} = (118,500 \text{ lux}) \left\{ 1 + 0.034 \cos \left[ \frac{2\pi}{365} (J - 2) \right] \right\} \exp \left( -\frac{0.21}{\sin a_t} \right) \quad (1)$$

(DiLaura 1984). In Equation (1), the solar illumination constant is taken to be 118,500 lux,  $J$  is Julian date, and  $a_t$  is the solar altitude angle.

A number of assumptions and simplifications facilitate the analysis of the transfer of the flux in the solar beam to a room. The solar beam's spread of approximately half of a

degree is ignored, so that the cross-sectional area assumed for the reflected beam does not change with distance from the reflector. If  $E_{dn}$  is taken as the average flux divided by the value of this cross-sectional area, then the flux leaving the reflector's surface in the reflected beam is the same as the incident flux except for losses due to reflection and due to transmission through glazings.

According to the geometric circumstances shown in Figure 1, the cross-sectional area that contains the flux leaving the surface of the reflector is  $(S \cos B)$ . If net reflectance is designated as  $\rho_r$  and net transmittance through the glazings is  $T_g$ , then the flux leaving the reflector's surface in the reflected beam is given by the quantity  $F_r$  having a value of

$$F_r = T_g \rho_r E_{dn} S \cos \beta \quad (2)$$

The flux  $F_r$  is projected to an area on the ceiling that is labelled  $S'$  in Figure 1. The location and shape of area  $S'$  are determined by the direction of the beam sunlight incident on the reflector, the size and orientation of the reflector, and the position of the reflector relative to the ceiling. As long as there are no additional sources or losses between the reflector and the ceiling, the quantity of flux available for work plane illumination can be determined by the approximation derived below, without knowing the location, shape, or size of area  $S'$  on the ceiling.

The fraction of the flux that is reflected from the ceiling is determined principally by the reflectance of the ceiling.

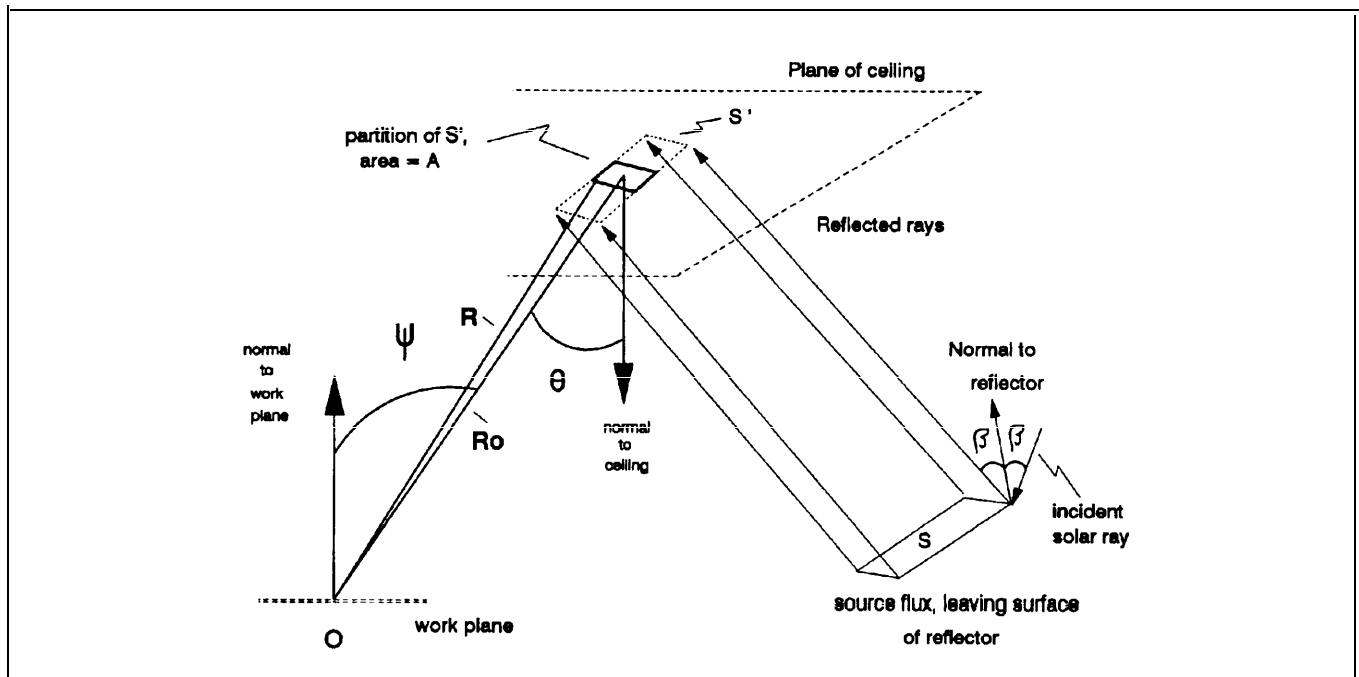


Figure 1. Geometry for Deriving an Illuminance Model

The ceiling is taken here to be a fully diffuse Lambertian reflector, whose reflected luminance is independent of direction. Let the reflectance of the ceiling be  $\rho_c$ . Due to its assumed Lambertian nature, the total flux leaving the illuminated patch of area  $S'$  on the ceiling is:

$$F_c = \rho_c F_r \quad (3)$$

Subdivisions of the area shown as  $S'$  in Figure 1 can be taken, and if they are small enough, they can be treated as if they are point sources viewed from a point in the work plane. One such subdivision of  $S'$  is shown in Figure 1 as a square patch of area  $A$ , and the center of the square patch is at a distance  $R_o$  from the work plane. The patch can be treated as a point source if its area is small enough so that  $A \ll R_o^2$ .

The total illuminance (or exitance)  $E$  emitted by a Lambertian source of luminance  $L$ , into a full hemispherical solid angle of  $2\pi$  steradians, is given by  $E = \pi L$ . To a first approximation, the average luminous exitance reflected from the ceiling is the flux  $F_c$  divided by the area  $S'$ . The average reflected luminance ( $L_o$ ) is then the value of the average exitance divided by  $\pi$ , or,

$$L_o \approx \frac{F_c}{\pi S'} \quad (4)$$

The illuminance at point O due to the patch of area A depends on the three quantities shown in Figure 1 as  $R_o$ ,  $\theta$ , and  $\psi$ .  $\theta$  is the angle between the line  $R_o$  and the direction normal to the ceiling, and  $\psi$  is the angle between the line  $R_o$  and the direction normal to the work plane. Because the luminance is assumed to be uniformly distributed throughout area  $S'$ , luminance within the patch of area A will be  $L_o$  as given by Equation (4). The expression for the illuminance at O due to the patch of area A is given as:

$$E_o = \int_A L_o \frac{\cos\theta \cos\psi da}{R_o^2} \quad (5)$$

$$\approx \frac{F_c A \cos\theta \cos\psi}{\pi S' R_o^2}$$

The assumption of  $A \ll R_o^2$  implies that the angular quantities and the distance  $R_o$  do not change by much over the entirety of area A. This allows all of the quantities inside the integrand of Equation (5) to be removed as constants. The expression for  $L_o$  as given by Equation (4) is substituted to give the form of  $E_o$  in Equation (5).

In most rooms, the plane of the ceiling is parallel to the work plane, i.e.,  $\theta = \psi$ . Combining Equations (2), (3), and (5) with this fact, the final form of  $E_o$  can be expressed as:

$$E_o = \rho_c T_r \rho_r \times \frac{A E_{dn} S \cos\beta \cos^2\theta}{S' \pi R_o^2} \quad (6)$$

Let the distance from any point on the boundary of the partition to point O be called  $R$ , as shown in Figure 1. For purposes of calculation, the following constraint can be used to determine the extent of partitioning of the area  $S'$ :

$$0.95 R_o \leq R \leq 1.05 R_o \quad (7)$$

In other words, a 5% variation in  $R_o$  is tolerated. The location and shape of  $S'$  on the ceiling can be found by knowing the locations of the corners of the reflector of area S in the room's coordinate system and the direction of the reflected light (Stiles 1992).

The first use of the model Equation (6) was in a comparison of computed illumination levels with actual photometric measurements taken in a classroom with a SPRA retrofit. Computed levels were within about 100 lux of the measured levels at corresponding positions in the room. Repeated field measurements of illumination at a single position in a room over a brief time can vary by about 100 lux (Stiles 1992b). The computed levels were thus within the precision of the field measurements.

The fact that computed illuminances were within the range of roughly measured values suggests that the model derived above captures the magnitude of work plane illumination from SPRA's flux to a first order. Additional phenomena, such as the contributions of multiple reflections of SPRA's flux throughout the room, are apparently second-order considerations and will not be considered further here.

## Simulations of Illuminating Performance

### Simulation Parameters

Figure 2 illustrates the spatial parameters of the floor plan of the room assumed for solar lighting simulations. The south-facing room is square, 9.14 m (30') to a side, with four windows evenly spaced on the south wall. Each window provides a square area, with 0.91 m (3') to a

side, for SPRA hardware. Ceiling height is 3.05 m (10') above the floor.

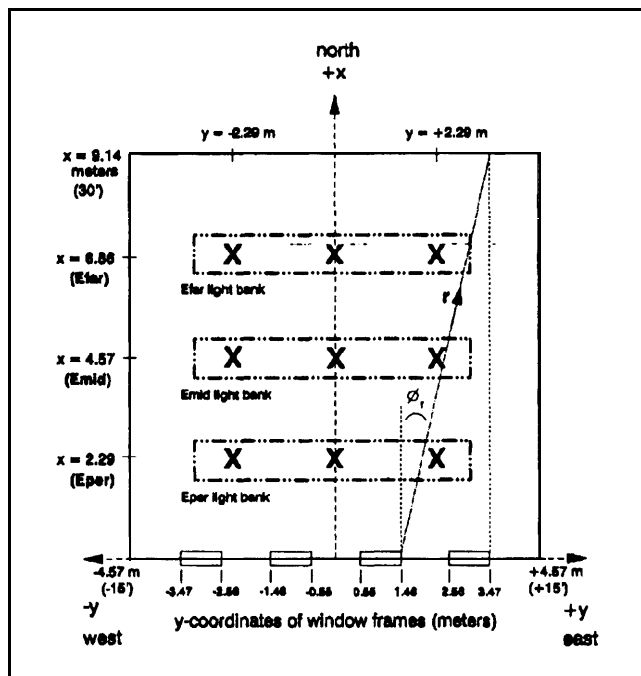


Figure 2. Model Room Plan and Coordinate System

The coordinate system superimposed on the floor plan in Figure 2 is marked with the locations of the windows and of the nine points for which illumination levels were computed in the simulations. The nine points are located in a desk-level work plane, 0.91 m (3') up from the floor. The rows of the sampling grid are seen to be parallel to the window wall. At 2.29 m (7.5') from the windows, the illuminances of the perimeter are defined to occur in the row labelled  $E_{per}$ . Similarly, the illuminance sampling points in rows 4.57 m (15') and 6.86 m (22.5') from the window wall are labelled rows  $E_{mid}$  and  $E_{far}$ , respectively.

Simulation illuminance values were calculated at the nine positions in a grid whose rows correspond to those shown in Figure 2. The fluorescent lights in the simulation were distributed in three banks, each parallel to the window wall. The positions of these light banks are indicated in Figure 2.

The size of the simulated room is typical of the educational and commercial spaces that we have encountered in our design work (Stiles and Kinney 1991). The values assigned to the reflectance and transmittances were taken from nominal design values for daylighting (Kaufman and Christensen 1984). Diffuse ceiling reflectance was taken as 0.8, net transmittance of the double pane windows was 0.64, and specular reflectance of the SPRA reflectors was taken as 0.9.

In addition to the simulated illuminances provided by the SPRA systems, natural daylight levels were also computed by IESNA's Lumen Method for Sidelighting (Kaufman and Christensen 1984; Libbey-Owens-Ford Company 1976). Clear sky conditions were assumed in all cases. As such, there would be significant glare from the direct solar beam through the windows in many areas of the room. Therefore, for purposes of simulating base-case illuminances without SPRA, it was assumed that occupants would draw shades over all window areas during daylight hours. The net transmittance of the shades was taken to be 0.2.

Ninety percent of each window area was taken to be available for transmission of daylight. Wall and floor reflectance were assumed to be 0.7 and 0.3, respectively. Base-case sources of illumination were taken to be direct beam and diffuse skylight. No contributions were included from ground-reflected illumination.

A south-facing window wall was assumed, because a study of SPRA performance as a function of window wall direction was beyond the scope of the present paper. Three geographic regions of the United States were studied, however. A northern region, a median region, and a southern region were selected at latitudes of 45°, 35°, and 25°, respectively.

The illuminances in the room were computed at increments of 15 days, from January 10 through the end of the year. Illuminances with and without SPRA contributions were computed, based on the simulation parameters outlined above. On each of these days, average illuminance levels were computed for half-hour intervals between the times of 9 solar hours and 15 solar hours. No attempts were made to translate solar time into local time. It was assumed that even during the months of daylight savings time, the room would be occupied between 9 and 15 solar hours. Experience has shown that before and after this interval, reflections from the SPRA systems are too close to the windows to provide adequate illumination deeply in the room.

A representation of the SPRA systems in a window is given by the sketch in Figure 3. In that figure, a view of the two arrays in each window is provided, one on the left and one on the right, from the perspective of someone inside of the room. Note that the positive z-axis has its origin at the floor and extends to a value of  $z = 3.05$  m (10') at the ceiling. Also, the simplified rendition of the SPRA installation shown in Figure 3 has only three reflectors in each array; in the simulations, there were ten reflectors per array.

According to previously described design criteria (Stiles 1992a; 1992b), the array on the left side of the window as

shown in Figure 3 is the *am* array, in that it reflects the morning sunlight into the room. The array on the right side is the *pm* array, which reflects the afternoon sunlight into the room. Both of the arrays reflect sunlight into the room between the solar hours of about 11-13. At solar noon, the azimuth angle of the sun is perpendicular to the window wall.

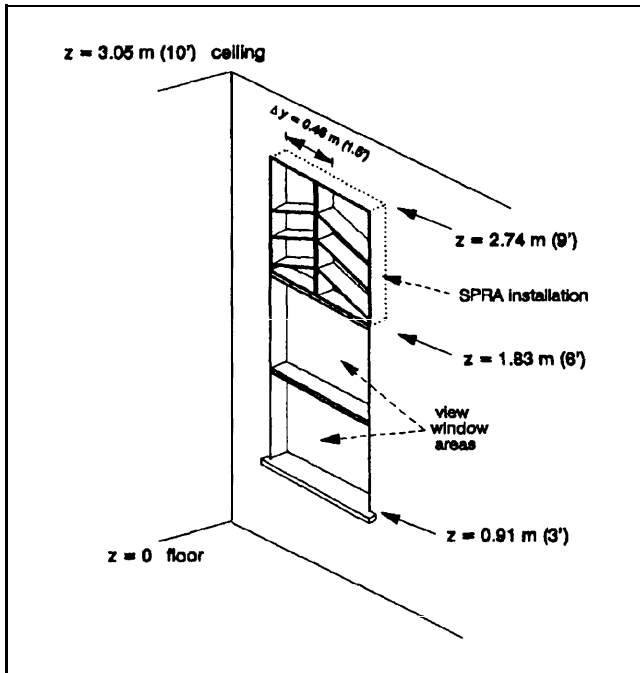


Figure 3. Sketch of Reflector Arrays in a Window

It is assumed that the areas of the upper window not occupied by the SPRA daylighting reflectors are opaqued. This is to prevent direct beam or diffuse sky light from entering through those areas. The close spacing of the reflectors, their pitched dispositions, and their own opacity make the arrays roughly analogous to venetian blinds. Indeed, SPRA systems have been observed to block direct beam sunlight (Stiles 1992b).

For purposes of simulations, it was assumed that the only light admitted to the room through SPRA was the reflected beam, which produced a work plane illuminance defined by Equation (6). Diffuse skylight contributions were ignored, principally because SPRA does not project the rather weak diffuse reflections for more than a few feet from the window wall (Stiles 1992b).

There are a number of design features common to all of the arrays in the simulations for the three geographic regions. These features have been found to simplify and make systematic the fabrication of SPRA systems and the analysis of their performance:

1. The reflected solar flux is projected to the ceiling.
2. The orientations of the reflectors are selected so that solar reflections *never project downward into the room* (Stiles 1992a; 1992b; 1994), a feature that eliminates the possibility of line-of-sight glare from the systems.
3. All of the reflectors in a given array have the same orientation, so that the reflections are parallel.
4. All reflectors occur between the right and left sides of the frame for the array, with no attachments at the top or the bottom of the frame; all reflectors in an array thus have the same length. This arrangement greatly simplifies the design and assembly of an array.
5. The minimum distance between the top and bottom reflectors and the top and bottom of the frame, respectively, is 5.08 cm (2"). This provides sufficient clearance for maintaining structural integrity of the frame during the transport and the installation of the systems.
6. The positions of the reflectors are given in the same coordinate system used to describe the room (e.g., see the x- and y-axes in Figure 2).
7. The reflectors are assumed to be second-surface mirrors, 0.32 cm (1/8") thick and 5.08 cm (2") wide. This is a prevalent and inexpensive reflector, and bears installation well.

A very useful short-cut for specifying reflector orientations has been found for rooms of the dimensions assumed in our simulations. This short-cut can be thought of as a method for *limiting the angles of reflection* in the room. An example is provided for the *am* arrays in Figure 2.

The design day and time is at solar noon on the winter solstice, when reflected patterns achieve their greatest excursion from the window wall. Reflections from the eastern edge of one window are allowed to traverse an azimuth angle (Stiles 1992a),  $\phi'$ , of between eight and fifteen degrees. In the symmetric case shown in Figure 2, the reflection points toward the north wall at the same y-coordinate of the eastern edge of the window to the east of the window under consideration. The reflection azimuth angle is about 12 degrees.

At solar noon on the winter solstice, the *zenith angle of reflection* (DiLaura 1984; Stiles 1992a; 1992b) is selected so that the lowest reflector in an array projects its flux to the ceiling in the direction of the north wall. For the room geometry selected, the zenith angle of reflection turns out to be about 86 degrees (not shown). Reflector orientations selected in this way have limits on the depth of projection of their flux across the ceiling, the farthest extent of which occurs at solar noon on the winter solstice.

The components of the unit vectors perpendicular to the reflector surfaces in a previously described solar coordinate system are given in Table 1 for the am arrays at each of the three geographic locations (Stiles 1992a). These vectors describe the orientations of the reflectors. Because of the symmetry for south-facing rooms, the corresponding vectors for the reflectors in the pm arrays are the same except for the fact that their j components are negative.

**Table 1. Components of Reflector Orientation Vectors**

Geographic Location (deg latitude)	Vector Component		
	i	j	k
25°	0.2570	0.2401	0.9361
35°	0.1657	0.2926	0.9418
45°	0.0704	0.3755	0.9241

Programs were created for computing illuminance levels at each of the nine grid points in the work plane shown in Figure 2. Equation (6) was the basis for these calculations, and was constrained according to Equation (7).2

In order to translate room-illumination levels into reductions in electric lighting, a sensor-based control strategy was assumed. It was assumed that three generic sensor signals monitored the average illuminances of the respective rows of the sampling grid, with one sensor below each bank of electric lights (see Figure 2). Each bank of lights was thus controlled by one sensor, and the sensor's signal was assumed to be in a constant proportion to work plane illuminance. Further, the sensor for controlling each bank of lights was assumed to be shielded, positioned, and calibrated to keep electric light output within an appropriately selected range (Rubinstein 1984).

Savings computations were derived for fractional reductions in electric energy usage over half-hour intervals throughout the 6-hour period for which the SPRA systems contributed to daylighting levels each day. If the input power to the j<sup>th</sup> bank (row) of electric lights is P<sub>j</sub> and is given in kiloWatts, and if an interval of time Δt is given in hours, then the energy consumption of that bank of lights in the i<sup>th</sup> time interval is P<sub>ji</sub>Δt, in kWh. Because there are three banks of lights assumed for the room, the total lighting energy consumption for the room over a 6-hour period can be given as:

6-hour kWh consumption =

$$\sum_{j=1}^3 \sum_{i=1}^{12} P_{ji} \Delta t = \Delta t \sum_{j=1}^3 \sum_{i=1}^{12} P_{ji}, \tag{8}$$

$$\Delta t = 0.5 \text{ hrs}$$

If all of the lights are turned on during the 6-hour period in question, maximum consumption occurs. In this case, all of the P<sub>ji</sub> can be assumed to be equal to a constant, P<sub>o</sub>. Under these conditions, Equation (8) assumes a value of (36 P<sub>o</sub> Δt) kWh.

If the level of electric light output of the j<sup>th</sup> light bank is modulated in a control strategy in the i<sup>th</sup> time interval, the input power can be expressed as a fraction, α<sub>ji</sub>, of the quantity P<sub>o</sub>,

$$P_{ji} = \alpha_{ji} P_o, \tag{9}$$

$$0 \leq \alpha_{ji} \leq 1$$

Equation (10) gives the actual lighting energy used during a 6-hour period if a conservation strategy is employed:

$$\Delta t \sum_{j=1}^3 \sum_{i=1}^{12} (\alpha_{ji} P_o) = P_o \Delta t \sum_{j=1}^3 \sum_{i=1}^{12} \alpha_{ji} \tag{10}$$

The fractional usage of the lights is found by dividing Equation (10) by the value of the maximum consumption, (36 P<sub>o</sub>Δt). The difference between unity and the fractional usage is the fractional savings. The percentage of savings in the 6-hour interval can then be given by Equation (11):

$$\% \text{ savings} = \left( 1 - \frac{\sum_{j=1}^3 \sum_{i=1}^{12} \alpha_{ji}}{36} \right) \times 100\% \tag{11}$$

The problem becomes one of relating the illuminances at the sampling grid locations with the α<sub>ji</sub> terms. This was done by taking the average of the computed illuminance values in each row of the sampling grid. Thus, the average of the lux values in the E<sub>row</sub> was the "sensed" quantity for bank j = 1, and this average was a quantity

$E_{j=1}$ . The averages of the values in the  $E_{mid}$  and the  $E_{far}$  rows were the sensed quantities for the other two banks, giving average sensed levels of  $E_{j=2}$  and  $E_{j=3}$ , respectively.

The required illumination level at the work plane was taken to be 750 lux, which is rather stringent, but representative of the mid-range levels recommended for educational and commercial spaces (Kaufman and Christensen 1987). A lower required level would result in a higher average level of savings from conservation strategies. As such, the savings computations described below constitute a worst-case performance for the daylighting strategies.

Two conservation strategies were studied. The first was an *on-off* strategy for electric lighting control, defined quantitatively as the following set of conditions:

$$\alpha_{ji} = \begin{cases} 1 & \text{if } E_j < 750 \text{ lux} \\ 0 & \text{if } E_j \geq 750 \text{ lux} \end{cases} \quad (12)$$

The second strategy was one for *dimming* the fluorescent lights in order to maintain a net illuminance of 750 lux (if daylighting levels did not exceed 750 lux). A simplified approach to this rather complicated technical problem was adopted. First, a maximum output of 1000 lux at work plane level was assumed for the electric lights. Second, a linear average of a typical power curve (Kaufman and Christensen 1984) was derived for the range of 1-750 lux of output. This transfer function was taken to be: <sup>3</sup>

$$\% \text{ of power input} = 0.067[\text{output in lux}] + 15 \quad (13)$$

According to Equation (13), an output of 750 lux requires about 65% of the maximum power input. Based on these assumptions, the fractional power in the dimming control strategy for electric lights becomes:

$$\alpha_{ji} = \begin{cases} 0 & \text{if } E_j \geq 750 \text{ lux,} \\ \frac{0.067(750 - E_j) + 15}{65} & \\ \text{if } 1 < E_j < 750 \end{cases} \quad (14)$$

In each case of control for the electric lights, savings were computed under two conditions. First was the condition in which the SPRA systems did not contribute to work plane illumination, which defines the base-case. Second was the

condition in which the base-case illumination levels were summed with the contributions from the SPRA systems.

## Results

**Illuminance Levels.** Figure 4 shows representative illuminance levels *due to the SPRA systems alone* at summer solstice and at winter solstice, for the median latitude. Results (in lux) at 9 solar hours and at solar noon are shown, and the numbers are presented for the grid locations defined in Figure 2. Note that due to the symmetries, the results at 15 solar hours are the transpose of those from 9 hours about the middle column of the numbers.

Site Latitude: 35 deg N Illuminance levels (lux), from SPRA only										
	Solar Time: 9 hrs.				Solar Time: 12 hrs.					
Julian Date:	10	10	10	Efar	310	370	310			
355 (winter solstice)	50	50	40	Emid	370	430	370			
	220	230	190	Eper	180	200	180			
Julian Date:	20	20	20	Efar	40	40	40			
175 (summer solstice)	60	70	60	Emid	150	170	150			
	310	350	320	Eper	790	900	790			

Figure 4. Examples of Illuminance Levels from SPRA

Perimeter light levels are elevated by 200-300 lux at 9 solar hours throughout the year, with the higher levels near summer solstice. At 9 hours, the arrays contribute less than 100 lux to positions in the remainder of the room.

The best performance of the SPRA system is seen at solar noon on the winter solstice, when 300-400 lux are supplied up to about 7 m (23') from the window wall. At solar noon on the summer solstice, the projected illumination does not reach as deeply into the room, but perimeter light levels are elevated in the 800-900 lux range.

Qualitatively similar temporal distributions of illuminance are found in the simulations for the northern and southern latitudes. *In their worst performances, the SPRA systems offer significant perimeter lighting throughout the year, within about 3 m (10') of the window wall. At their best performances, these systems can elevate core room levels by 300-500 lux for a few hours every clear day near winter solstice.* Quantitative estimates of savings in electric lighting can be made from the detailed time series of simulations throughout the year.

**Savings in Electric Light Usage.** The percent savings in electric lighting energy due to the various daylighting scenarios are plotted against Julian date in Figures 5-7. cursory examination of the plots reveals these similarities:

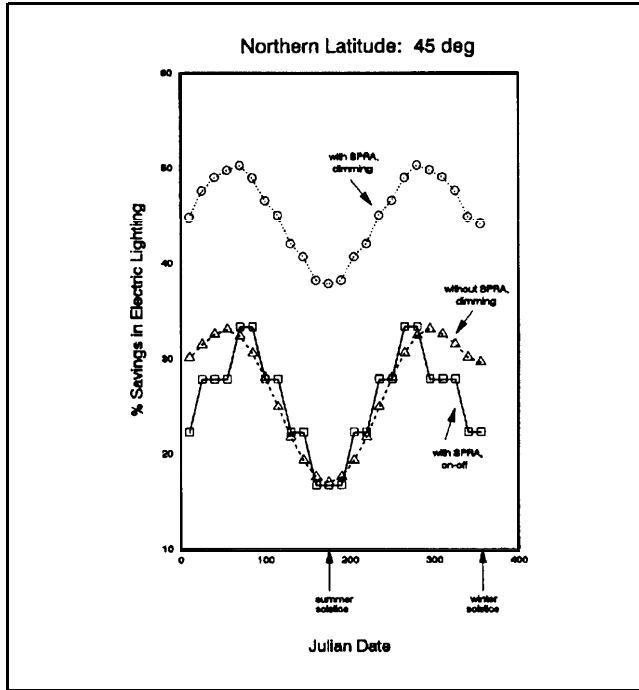


Figure 5. Savings in the Northern Latitude

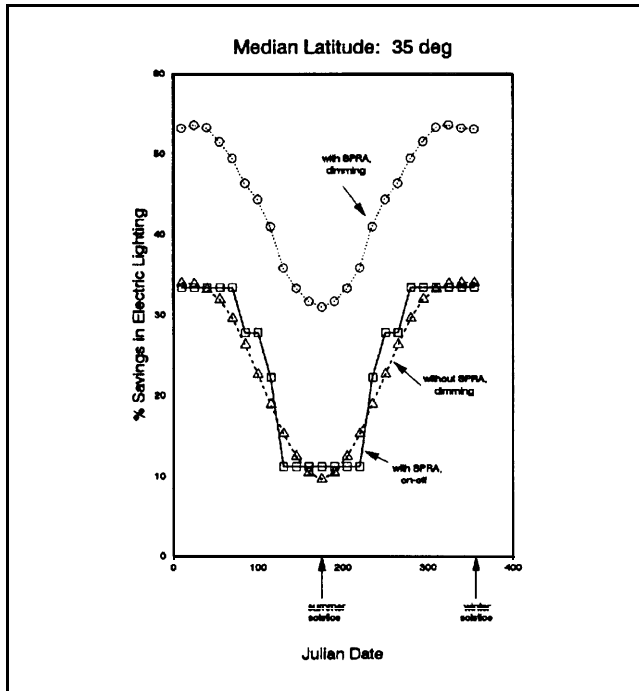


Figure 6. Savings in the Median Latitude

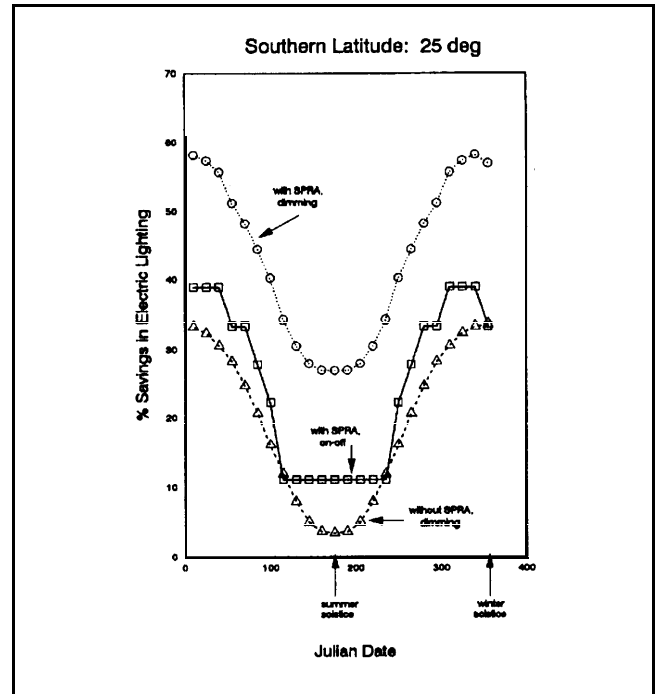


Figure 7. Savings in the Southern Latitude

- There were *no* computed savings throughout the year in the case of on-off electric lighting control without SPRA. The shades that prevent glare from the daylighting windows when SPRA reflectors are not present reduce daylight levels in the room so much that these levels are never sufficient to turn off the electric lights.
- The timing and magnitudes of savings for the cases of on-off electric control with SPRA, and dimming control without SPRA, are very comparable.
- The greatest savings result from the combination of dimming electric control with SPRA.
- The minimum savings occur for all room treatments near the summer solstice.
- In each case of electric lighting control, the SPRA systems enhance savings principally in the times between the summer solstice and the equinoxes.

There are a number of differences as well. The savings at the northern latitude peak near the equinoxes (Figure 5), whereas peak savings occur closer to the winter solstice at the median and southern latitudes (Figures 6 and 7, respectively). The disparity (range) between maximum and minimum savings increases from north to south for all three plotted cases of treatments. At the southern latitude, the SPRA/on-off treatment produces slightly greater savings than the no SPRA/dimming treatment.



Many of the seasonal differences in savings performance can be explained on the basis of solar altitude. The lower the sun is in the sky, the greater the distance the reflected beam extends into a south-facing room. The deeper the reflected rays go across the ceiling into a room, the higher are the light levels farther from the window, and there are opportunities for reducing electric lighting at the middle and far banks of lights.

Average annual savings were computed from the data plotted in Figures 5-7. This information is given in Tables 2, 3, and 4 for the northern, median, and southern latitudes, respectively. Despite the increases in the ranges of values from north to south, annual averages for any given treatment are remarkably consistent, varying by only a few percentage points for most treatments. The greatest variation in annual average savings is for the no SPRA/dimming treatment, but the variation ranges only from about 27% in the north to about 19% in the south. In the cases of SPRA daylighting, the dimming control strategy almost doubles the savings obtained with the on-off control strategy throughout the country.

**Table 2. Average Savings, Northern Latitude**

Type of Daylighting	Type of Electric Lighting Control	Average Annual Percent Savings	Range
Without SPRA	on-off	0	--
With SPRA	on-off	25.7	16.7-33.3
Without SPRA	dimming	27.1	17.0-33.1
With SPRA	dimming	45.3	37.8-50.2

**Table 3. Average Savings, Median Latitude**

Type of Daylighting	Type of Electric Lighting Control	Average Annual Percent Savings	Range
Without SPRA	on-off	0	--
With SPRA	on-off	25.0	11.1-33.3
Without SPRA	dimming	24.1	9.6-33.9
With SPRA	dimming	44.5	30.9-53.5

**Table 4. Average Savings, Southern Latitude**

Type of Daylighting	Type of Electric Lighting Control	Average Annual Percent Savings	Range
Without SPRA	on-off	0	--
With SPRA	on-off	25.0	11.1-38.9
Without SPRA	dimming	19.4	3.5-33.6
With SPRA	dimming	43.0	26.9-58.1

## Conclusions

A model of room illumination due to specular reflectors at windows was derived, and it proved trustworthy when compared to preliminary field measurements of illuminance increases due to SPRA. Its principle use in this paper was for simulating illumination levels from SPRA systems throughout the year. These simulations applied to a period from three hours before to three hours after solar noon during a typical workday in a south-facing commercial or educational room that is 9.14 m (30') square. Knowing the illumination levels from daylighting sources, it was possible to simulate the potential savings in electric lighting energy. A rather stringent requirement for work plane illumination was imposed (750 lux). There would have been higher computed savings for lower illumination requirements.

There are additional factors that can be considered in assessing the veracity of the simulations. The prevalence of cloud cover was not taken into account. The trends in savings shown in Figures 5-7 assume clear skies, which tend to be less prevalent in the northern regions. Although not a part of the present study, reductions in electric lighting can be expected to save on air conditioning loads. This is a factor that bears further consideration for southern regions. The savings projected for the southern areas in Figure 7 thus may underestimate the total available savings there.

All other factors being equal, there were some notable results obtained from the simulations. It was assumed that the occupants of a room would draw shades over all window areas on a bright, sunny day, a condition that showed no savings when an on-off control strategy for electric lights was applied without SPRA. *Simulations showed that it was SPRA that created the potential for savings in electric lighting under such circumstances.* This is because SPRA usefully reflects solar flux onto the ceiling, glare-free, yet provides work plane illuminance.

Simulations of a dimming control strategy for the electric lights suggested that SPRA can supplant almost half (44%) of the lighting needs. The respective average percentages of savings for both lighting controls were remarkably consistent in the cases of all three latitudes examined.

There was a finding that dimming electric control without SPRA achieves roughly the same savings as on-off control with SPRA. This implies that either treatment may be selected for about a 25% average savings. Initial costs for both types of treatments may be comparable, but the continuous dimming strategy requires a rather careful calibration to ensure that sensor output modulates fluorescent light levels in the appropriate range (Rubinstein 1984). This complication may make the option of the SPRA/on-off treatment more attractive at set-up time.

It is informative to roughly estimate the monetary savings due to SPRA combined with the simpler on-off control strategy for electric lights. In a typical working year, there are about 2,000 hours of working space occupancy, 6/8 of which are available for SPRA daylighting. A typical office or classroom of the dimensions given in Figure 2 uses about ten 100-watt lighting fixtures, or about a kiloWatt of lighting power (Kaufman and Christensen 1987; Stiles 1992b). At  $(6/8) \times (2,000)$  hours of operation, electric lighting consumption is about 1,500 kWh. A savings of 25% of this energy amounts to about 375 kWh. Given a ballpark estimate of about ten cents per kWh, the savings of 375 kWh implies an annual savings of about \$37.

Arrays of the type that are illustrated in Figure 3 require an average of about  $0.5 \text{ m}^2$  ( $5 \text{ ft}^2$ ) of reflecting material per window (the majority of the cost for the arrays). Common 8-inch mirror costs about \$3 per square foot at the writing of this paper, or about \$15 of reflector per window. At four windows per room, \$60 worth of reflector is needed.

If the SPRA systems save about \$37 a year, the payback for the \$60 of materials for the arrays is about two years. If the cost of the arrays is marked up by 200% for production and installation, payback for materials is still within four years, from savings in lighting energy. Counting the cost of the on-off control systems, a payback period of about five years is not unreasonable. This makes SPRA competitive as a technology for enhancing savings with electric lighting controls.

We are continuing to explore refinements of methods for distributing the flux from SPRA systems more uniformly in a room. We also plan on studying the use of daylighting from SPRA to reduce air conditioning loads. However, the seminal findings of our present work suggest that the SPRA technology can save significantly on

electric lighting throughout the sunny regions of the United States.

## Endnotes

1. In many cases, the entire perimeter of the area  $S'$  itself met the constraints of Equation (7). In such cases, the limit of integration in Equation (5) becomes  $A = S'$ , and the terms  $A$  and  $S'$  cancel each other in Equation (6).
2. One circumstance that was noted was the occasional *overlap* of patterns on the ceiling, especially at times near the winter solstice when projections are longest on the ceiling. However, it was found that the thin patterns often overlapped by only 10-15% of their total areas, a matter of 4-60  $\text{cm}^2$  of the hundreds of square cm of a given reflector's pattern. To a first approximation, overlapping areas were ignored in the calculations.
3. The transfer function was derived from information in Figure 8-46 of Kaufman and Christensen (1984).

## References

- Benton, C. 1986. "Field Measurements of Light Shelf Performance in a Major Office Installation," *Proc. 11th National Passive Solar Conference*, American Solar Energy Society.
- DiLaura, D. L., Chairman of the IES Calculation Procedures Committee. 1984. "Recommended Practice for the Calculation of Daylight Availability." *Journal of the Illuminating Engineering Society*, July: 381-392.
- Kaufman, J. E., and Christensen, J. F., eds. 1984. *Illuminating Engineering Society Lighting Handbook Reference Volume*. Illuminating Engineering Society of North America, New York.
- Kaufman, J. E., and Christensen, J. F., eds. 1987. *Illuminating Engineering Society Lighting Handbook Application Volume*. Illuminating Engineering Society of North America, New York.
- Libbey-Owens-Ford Company. 1976. *How to Predict Interior Daylighting Levels*. LOF, Toledo, Ohio.
- McCluney, R. 1983. "Passive Optical Solar Tracking System." *Applied Optics*, 22:3433-3439.
- McCluney, R. 1994. *Introduction to Radiometry and Photometry*. Artech House Publishers, Norwood, MA.

Rubinstein, F. 1984. "Photoelectric Control of Equi-Illumination Lighting Systems." *Energy and Buildings*, 6:141-150.

Stiles, M., and L. Kinney. 1991. *Design, Development and Evaluation of Innovative Daylighting Strategies, Phase II Final Report*. Synertech Systems Corporation Technical Report SYN TR 91-525, Syracuse, NY.

Stiles, M. 1992a. "Optimal Beam Daylighting with Stationary Projecting Reflector Arrays," *Proc. ACEEE 1992 Summer Study of Energy Efficiency in Buildings*, Volume 1, pp. 1.225-1.235. American Council for an Energy Efficient Economy, Washington, D.C.

Stiles, M. 1992b. *Optimal Design, Fabrication and Final Prototype Testing of Stationary Projecting Reflector Arrays*. Synertech Systems Corporation Technical Report SYN TR 92-536, Syracuse, NY.

Stiles, M. 1993. "Fundamental Directional Properties of Active and Passive Beam Daylighting Systems," *Proc. Canadian Solar Energy Society 1993 Summer Conference*, Quebec, Canada.

Stiles, M. 1994. "Apparatus and Method for Optimizing Useful Sunlight Reflected into a Room," U.S. Patent 5,285,315.